Multi-objective Optimization Problems for Portfolio Selection: The MDRS Model

1 Introduction

The *portfolio selection problem* is one of the so-called multi-objective optimization problems, in which an investor desires to know how her/his capital should be allocated between the different assets available in the market in order to guarantee the maximum return of the investment at the minimum risk. The multi-objective nature of these problems is unquestionable and, since the classical mean-variance optimization problem of Markowitz was proposed in [12], portfolio theory has evolved considerably. Nowadays more complex and sophisticated models are used. Some of them include more than two criteria to find Pareto optimal portfolios [2, 15, 20, 23], thus making the problem more difficult to solve. Also, instead of the variance of the returns, alternative statistical measures of the risk have been employed, such as e.g. the semi-variance, the absolute semi-deviation or the value-at-risk (see [8, 19] and references therein). Furthermore, to make it more realistic, information about trading, transaction costs, specific requirements of the investor, etc. can be also considered in the model. To this end, additional constraints are introduced, such as e.g. cardinality constraints (to limit the number of assets participating in the feasible portfolios), lower and upper bound constraints (to set lower and upper limits for the amount invested in each asset), and so on.

However, in portfolio selection, deciding the approach to quantify the uncertainty of the portfolio returns is as important as determining the optimization model to be solved. In classical problems, the expected returns on assets are considered as problem parameters and are estimated throughout historical data sets assuming that the vector of returns on assets is multivariate-normally distributed. But financial information is not always completely available and, commonly, decisions are made under uncertainty. Then, a more realistic approach is to assume that the uncertainty of the future returns on the individual assets can be quantified by means of fuzzy numbers, which allow the introduction of the imperfect knowledge about the future market behaviour into the model [2, 10, 22]. Furthermore, instead of assuming that the uncertainty of the returns of the individual assets is approximated by fuzzy quantities, the uncertainty of the future returns of a given portfolio can be directly approximated by fuzzy numbers using the historical returns on the portfolio [1, 17, 20, 21].

Under the scheme, in [20], possibility distributions of LR fuzzy numbers are used to quantify the uncertain returns on a given portfolio, instead of using the combination of uncertainties provided by the returns of the assets that compose the portfolio. The membership functions of the LR fuzzy numbers defined are built using sample quantiles information from the historical data of the returns. Besides, [20] proposed a multi-objective optimization model for the portfolio selection problem, called the possibilistic *Mean-Downside Risk-Skewness* (MDRS) model, which is explained hereafter. The objectives considered are the return expected value of the portfolios (to be maximized) and two measures of the risk: the skewness of the future returns (to be maximized), and the absolute semi-deviation below the mean or the downside risk (to be minimized). Note that [20] introduced the skewness in order to incorporate a measurement of the asymmetry of the fuzzy return on a given portfolio and to study its role in the possibilistic portfolio selection problem. The MDRS model also includes bound and cardinality constraints for achieving both the diversification of investment and the control of the number of assets that compose the portfolios.

2 The MDRS multi-objective optimization model for portfolio selection

Next, we describe the MDRS model proposed in [20]. Let us consider a capital market with N financial assets offering uncertain rates of returns. An investor desires to know which is the optimal allocation of their wealth among the N assets, looking for the maximization of the expected return of the investment at the end of the period at the minimum risk. Denote a portfolio by $\mathbf{x} = (x_1, \ldots, x_N)^T$, where x_i represents the fraction of the total investment devoted to the asset i, for every $i = 1, ..., N$. This means that, if $x_i \neq 0$, the portfolio $\mathbf{x}$ invests in the asset i and the value of x_i indicates the corresponding proportion of the capital budget allocated to the asset i.

2.1 Quantifying the uncertainty

As already mentioned, [20] approximates the uncertainty of the return on a given portfolio directly instead of using the combination of uncertainties provided by the returns of the assets that participates in the portfolio, using sample quantiles information from the historical data.

For a portfolio $\mathbf{x}$, let us consider the historical returns over T quotation periods, denoted by $\{r_t(\mathbf{x})\}_{t=1}^T$, whose sample percentiles are given by p_j, being j the order of the percentile. The uncertainty regarding the future return on the portfolio $\mathbf{x}$ is approximated by a fuzzy number $\tilde{P}_{\mathbf{x}}$, whose membership function is built using these sample percentiles p_j. That is, the parameters that define $\tilde{P}_{\mathbf{x}}$ (core, spreads, and shapes) are obtained as functions of the sample percentiles of the historical dataset of the return on the portfolio $\mathbf{x}$. To be more precise, the uncertain return on the portfolio $\mathbf{x}$ is modelled by means of a bounded power LR fuzzy number $\tilde{P}_{\mathbf{x}} = \{p_l, p_u, c, d\}_{L_\pi, R_\rho}$, whose membership function is given by:

$$\mu_{\tilde{P}_{\mathbf{x}}}(y) = \begin{cases} L_\pi(\frac{p_l - y}{c}) & \text{if } p_l - c < y \leq p_l, \\ 1 & \text{if } p_l \leq y \leq p_u, \\ R_\rho(\frac{y - p_u}{d}) & \text{if } p_u \leq y < p_u + d, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where $[p_l, p_u]$ is the core (i.e. $([p_l, p_u] = \{y | \mu_{\tilde{P}_{\mathbf{x}}}(y) = 1\})$), c and d are the left and right spreads, while $\pi, \rho > 0$ are two positive real values known as shape parameters of the power reference functions L_π and R_ρ, respectively. The reference functions are defined as $L_\pi(t) = 1 - t^\pi$ and $R_\rho(t) = 1 - t^\rho$ and they are strictly decreasing and upper semicontinuous.

Since the fuzzy number $\tilde{P}_{\mathbf{x}}$ given by (1) defines a possibility distribution that matches with its membership function [24], power LR fuzzy numbers can be used to approximate the uncertain return on the portfolio $\mathbf{x}$, instead of aggregating the possibility distributions of the individual assets that compose $\mathbf{x}$.

Figure 1 shows the membership function of $\tilde{P}_{\mathbf{x}}$ for a given portfolio $\mathbf{x}$, with core $[p_l, p_u] = [-0.007, 0.006]$, spreads $c = 0.080$ and $d = 0.438$, and the shape parameters being $\pi = 0.538$ and $\rho = 0.228$, respectively. The points in the x-axis are the observed weekly returns on the given portfolio $\mathbf{x}$ over three years. Note the remarkable asymmetry of its membership function.

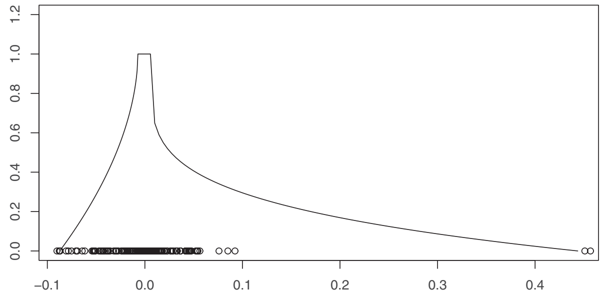

Figure 1: Plot of the membership function of a $\tilde{P}_{\mathbf{x}}$ built using the historical returns for a given portfolio $\mathbf{x}$.

2.2 Objective functions

Let us describe and introduce the explicit mathematical formulation of the objective functions of the MDRS model.

2.2.1 Possibilistic expected return value

To approximate the expected return value on a given portfolio $\mathbf{x}$, the concept of interval-valued expectation of LR fuzzy numbers is applied [7], as well as the usual defuzzification approach based on a crisp representation of the possibilistic moments. Thus, following [20], the possibilistic expected return value, denoted as $\bar{E}(\tilde{P}_{\mathbf{x}})$, is given by the middle point of the interval-valued expectation of the fuzzy number $\tilde{P}_{\mathbf{x}}$:

$$\bar{E}(\tilde{P}_{\mathbf{x}}) = \frac{p_u + p_l}{2} + \frac{d}{2}\frac{\rho}{\rho + 1} - \frac{c}{2}\frac{\pi}{\pi + 1}. \tag{2}$$

2.2.2 Possibilistic downside risk

Commonly, investors may be concerned with the risk of obtaining returns lower than the expected return value. Based on this observation, the possibilistic downside risk (i.e. the possibilistic absolute semi-deviation below the mean) is used to quantify the uncertain risk of the investment, since it penalizes negative deviations from the expected return, but not the positive deviations. For a fuzzy number $\tilde{P}_{\mathbf{x}}$ representing the uncertain return of a portfolio $\mathbf{x}$, the possibilistic downside risk, denoted as $w(\tilde{P}_{\mathbf{x}})$, is defined in [20] as follows:

$$w(\tilde{P}_{\mathbf{x}}) = p_u - p_l + d\frac{\rho}{\rho + 1} + c\frac{\pi}{\pi + 1}. \tag{3}$$

In general terms, it can be seen that $w(\tilde{P}_{\mathbf{x}})$ is evaluated as the length of the interval-valued absolute semi-deviation about the possibilistic mean value, that is, as the amplitude of the interval-valued expectation $E(\max\{0, \bar{E}(\tilde{P}_{\mathbf{x}}) - \tilde{P}_{\mathbf{x}}\})$.

2.2.3 Coefficient of possibilistic skewness

If the returns on the portfolios are known to be not symmetrically distributed around the mean, higher moments cannot be neglected to quantify the uncertainty on the return. A common measure of the asymmetry of the probability distribution of a random variable is the skewness, which is based on the third moment. A negative skew in the distribution of the returns indicates that they are not evenly distributed around the mean, having the right tail of the distribution shorter than the left tail. A positive skew means just the opposite, i.e. the left tail is shorter than the right one. Portfolios with a positive skew could be attractive to investors because extreme returns (those far from the expected return) are more probably associated with gains (if they are greater than the mean) than with losses.

Following [18], the coefficient of the possibilistic skewness of a fuzzy number can be calculated using the third possibilistic moment about the possibilistic expected value. Thus, for a fuzzy number $\tilde{P}_{\mathbf{x}}$ representing the uncertain return of a portfolio $\mathbf{x}$, the coefficient of the possibilistic skewness, denoted as $S(\tilde{P}_{\mathbf{x}})$, is defined as the ratio between the third possibilistic moment and the cube of the possibilistic standard deviation as follows:

$$S(\tilde{P}_{\mathbf{x}}) = \frac{\mu_3(\tilde{P}_{\mathbf{x}})}{w(\tilde{P}_{\mathbf{x}})^3}, \tag{4}$$

where $\mu_3(\tilde{P}_{\mathbf{x}})$ is the third possibilistic moment about the possibilistic expected value $\bar{E}(\tilde{P}_{\mathbf{x}})$ given in [18] as:

$$\begin{aligned} \mu_3(\tilde{P}_{\mathbf{x}}) &= \frac{1}{4}\left(d\frac{\rho}{\rho+1} - c\frac{\pi}{\pi+1}\right)^3 + \frac{1}{2}\left(d^3\frac{\rho}{\rho+3} - c^3\frac{\pi}{\pi+3}\right) + \\ &\quad + \frac{3(p_u - p_l)}{4} \\ &\quad \times \left[d^2\left(\frac{\rho}{\rho+2} - \frac{\rho^2}{(\rho+1)^2}\right) - c^2\left(\frac{\pi}{\pi+2} - \frac{\pi^2}{(\pi+1)^2}\right)\right] \\ &\quad - \frac{3}{4}\left(d^2\frac{\rho}{\rho+2} + c^2\frac{\pi}{\pi+2}\right)\left(d\frac{\rho}{\rho+1} - c\frac{\pi}{\pi+1}\right). \end{aligned} \tag{5}$$

According to (5), $\mu_3(\tilde{P}_{\mathbf{x}})$ is the middle point of the interval-valued third moment about the possibilistic expected value $\bar{E}((\tilde{P}_{\mathbf{x}} - \bar{E}(\tilde{P}_{\mathbf{x}}))^3)$. Note that the coefficient of possibilistic skewness is considered instead of directly using the third possibilistic moment in order to get a scale-independent measure of the asymmetry of the returns of the portfolios.

2.3 Constraints

Each portfolio $\mathbf{x} = (x_1, \ldots, x_N)^T$ must verify the budget constraint given as $\sum_{i=1}^{N} x_i = 1$, and the non-negative condition of every proportion, i.e. $x_i \geq 0$ for every $i = 1, ..., N$, when short selling is excluded.

Also, lower and upper limits on the budget to be invested in each asset i are imposed in the MDRS model to assure the diversification of the investment. For each $i = 1, 2, ..., N$, a bound constraint of the type $0 \leq l_i \leq x_i \leq u_i$ is considered, where l_i and u_i denote the lower and upper bounds for the asset i, respectively.

Additionally, a cardinality constraint is incorporated into the MDRS model to control the number of assets that participate in the portfolios. The cardinality constraint is given by $h_l \leq c(\mathbf{x}) \leq h_u$, where $c(\mathbf{x}) = \text{rank}(\text{diag}(\mathbf{x}))$ denotes the rank of the diagonal matrix whose diagonal elements are the components of the vector $\mathbf{x}$, and h_l and h_u are two positive integer values. Thus, the cardinality constraint ensures that the number of assets that compose each portfolio $\mathbf{x}$ is always within the interval $[h_l, h_u]$. Furthermore, if $h_l = h_u$, the portfolios are forced to be always composed by the same number of assets. Note that the function $c(\mathbf{x})$, which gives the number of positive proportions in the portfolio $\mathbf{x}$, is quasi-concave.

2.4 The MDRS model

Based on the aforementioned assumptions, the possibilistic *Mean-Downside Risk-Skewness* (MDRS) model is formulated as follows:

$$\begin{array}{llll} (\mathbf{MDRS}) & \max & \bar{E}(\tilde{P}_{\mathbf{x}}) & \\ & \min & w(\tilde{P}_{\mathbf{x}}) & \\ & \max & S(\tilde{P}_{\mathbf{x}}) & \\ & \text{s.t.} & \sum_{i=1}^{N} x_i = 1, & \text{(budget constraint)} \\ & & k_l \le c(\mathbf{x}) \le k_u, & \text{(cardinality constraint)} \\ & & 0 \le l_i \le x_i \le u_i, & \text{(bound constraints)} \\ & & \text{for } i = 1, 2, ..., N. & \end{array} \tag{6}$$

As explained, to calculate the explicit values of these objective functions, we need to know the quantiles of the historical returns of the portfolios to obtain the possibility distribution of $\tilde{P}_{\mathbf{x}}$. Thus, the MDRS model is a non-linear and non-convex multi-objective optimization problem due to the nature of quantiles calculation. Besides, the introduction of the cardinality constraint, with a quasi-concave function, means that the model is also NP-hard [14]. Dealing with a non-linear and non-convex NP-hard multi-objective optimization problem is not straightforward and classical multi-objective optimization methods are not suitable. Therefore, the use of heuristic approaches such as evolutionary multi-objective optimization algorithms is highly recommended for solving the above multi-objective optimization problem.

2.5 Data

To completely define the MDRS model, we need a case study with the financial historical data of the returns of a set of assets in a particular period of time. We can use the weekly returns on assets from the Spanish IBEX35 index. In particular, we have the historical data of $N = 33$ assets[1], observed in $T = 165$ periods (weeks) from January 2013 till March 2016. For every $i = 1, \ldots, 33$ and $t = 1, \ldots, 165$, the sample return on the individual asset i at week t, denoted by r_{ti}, is calculated as follows:

$$r_{ti} = \frac{CP_i(t+1) - CP_i(t)}{CP_i(t)}, \tag{7}$$

where $CP_i(t)$ denotes the closing price of the asset i on Wednesday at week t. Thus, for $t = 1, \ldots, 165$, the weekly return on each portfolio $\mathbf{x}$ for the week t is obtained as $r_t(\mathbf{x}) = \sum_{i=1}^{33} r_{ti} \cdot x_i$.

[1] Although this index is constituted by 35 assets, there were two assets which were not included in the IBEX35 index through all the time window considered.

With this, the membership function of the LR fuzzy number $\tilde{P}_{\mathbf{x}} = \{p_l, p_u, c, d\}_{L_\pi, R_\rho}$ representing the uncertainty of the future return on each portfolio $\mathbf{x}$ is built using the sample percentiles p_j of its weekly returns set, where j is the order of the percentiles. In particular, the core and the support of $\tilde{P}_{\mathbf{x}}$ are represented by the intervals $[p_l, p_u] = [p_{40}, p_{60}]$ (medium return values) and $[p_3, p_{97}]$ (the 3^{rd} the 97^{th} percentiles), respectively, while the shape parameters are obtained as $\pi = \ln(0.5)/\ln(\frac{p_{40}-p_{20}}{c})$ and $\rho = \ln(0.5)/\ln(\frac{p_{80}-p_{60}}{d})$, assuming that the sample percentiles p_{20} and p_{80} have a 50% possibility of being realistic (they are obtained in such a way that the fuzzy and empirical quartiles coincide).

In [20], they assume that the cardinality of the portfolios is restricted in the MDRS model to a given number $k \in \mathbb{N}$ (i.e. $k_l = k_u = k$), that is, the cardinality constraint in (6) is formulated as $c(\mathbf{x}) = k$. This means that every feasible portfolio is obligated to invest in exactly k of the N available assets. In practice, they set the cardinality in 9 assets ($k = 9$) following the advise of [3], which suggests that investors should not consider k-values above one third of the total number of assets because of dominance relationships. Also, they set the bound limits as $l_i = 0.0$ and $u_i = 0.2$ ($i = 1, \ldots, 33$) for assuring the diversification of the investment.

3 Evolutionary multi-objective optimization algorithms for solving porfolio selection problems

While classical portfolio optimization problems can be efficiently solved by applying classical optimization techniques, this is not the case if additional conditions, such as bound and cardinality constraints, are introduced. Mainly, the most significant difficulty is the generation of feasible portfolios satisfying the requirements imposed by the new constrains [11]. Additionally, the solution process required for finding a set of Pareto optimal or non-dominated portfolios is not trivial, specially in the presence of multiple objectives (three or more). In this regards, the usefulness of evolutionary multi-objective optimization (EMO) [4, 5] for solving constrained multi-objective portfolio selection problems is doubtless. In [11, 13], comprehensive literature reviews and recommendations for best practices about the use of EMO algorithms in portfolio selection problems are presented, which is a proof of the growing interest on this research field.

However, as concluded by [13], the majority of the research done in the EMO field for solving portfolio selection problems is focussed on models making use of only two objectives, being the expected return value and the variance the most commonly used objectives. Besides, according to [13], cardinality and bound constraints are mostly considered to define feasible portfolios.

Note that the majority of the works consider the expected return and the measures of the risk as known parameters. Applications of fuzzy numbers for quantifying the uncertainty of future returns on assets, using credibility or possibility distributions, can also be found in the literature. For example, [2] considered a fuzzy mean-variance-skewness model with cardinality and trading constraints and solved it by applying both fuzzy simulation and an elitist optimization genetic algorithm. [1] implemented a bi-objective optimization genetic algorithm for solving a fuzzy mean-downside risk model with cardinality and bound constraints, in which the approximation of the uncertain returns was done through trapezoidal fuzzy numbers. In [9], a multi-criteria credibilistic portfolio selection model was proposed, which maximized (short and long-term) return and liquidity, and considered the portfolio risk as a credibility-based fuzzy chance constraint. The fuzzy estimates were obtained assuming both trapezoidal possibility distributions and general functional forms. This model, which also included bound and cardinality constraints, was solved by applying a hybrid algorithm that integrated fuzzy simulation with a real-coded genetic algorithm. In [21], a fuzzy cardinality constrained bi-objective optimization model was solved by means of a genetic algorithm. They also use the fuzzy VaR in a post-optimization decision support stage to find Pareto optimal portfolios according to the investor preferences.

Concerning the MDRS model, [20] applied an evolutionary procedure specially designed for generating non-dominated portfolios of two alternative reformulations of the MDRS model. Each of these reformulations corresponds to a bi-objective optimization problem which optimizes two of the objectives of the MDRS model, while the third objective is considered as an additional constraint. With this, the authors analysed the influence of the skewness either as a criteria or as a constraint. The results obtained supported previous research in this regard: the introduction of the skewness as a goal provokes important changes in the Pareto optimal front of the portfolio selection problem, and consequently in the patterns of investment.

However, the genetic procedure developed in [20] was designed to manage bi-objective optimization problems and, therefore, the possibilistic MDRS model was not solved as a whole constrained multi-objective optimization problem. Later, [17] has solved the MDRS model optimizing the three criteria at the same time. They considered three EMO algorithms to find non-dominated portfolios: NSGA-II [6], MOEA/D [25] and GWASF-GA [16]. However, applying EMO algorithms to constrained portfolio selection problems requires a special care for handling the objectives and constraints [11]. In the MDRS model, the difficulty mainly comes from the cardinality constraint, since only a limited number of the available assets can participate in the feasible portfolios, but not all of them. To internally manage the constraints in the EMO algorithm, one option may be the use of common genetic operators with the application of a repair mechanism to guarantee the feasibility of the new portfolios generated [11]. Alternatively, [17] proposed new mutation, crossover and reparation operators designed ad-hoc for generating feasible portfolios according to the budget, bound and cardinality constraints included in the MDRS model. They showed that better non-dominated portfolios are obtained if the new operators are incorporated in the three EMO algorithms considered, in comparison to the results obtained by commonly used operators and the repair mechanism.

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