GECCO

Enhancing Partition Crossover with Articulation Points Analysis

Francisco Chicano, Gabriela Ochoa, Darrell Whitley, Renato Tinós

UNIVERSITAS MALACITANA
UNIVERSIDAD DE MÁLAGA

UNIVERSITY of STIRLING

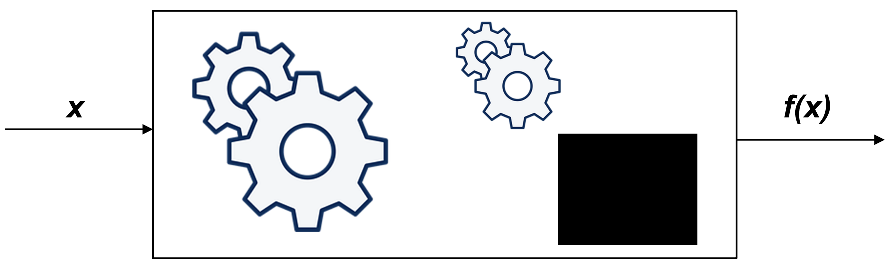
Colorado State University

UNIVERSIDADE DE SÃO PAULO
SCIENTIA VINCES

Outline

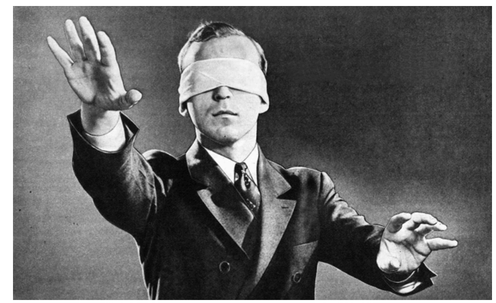
- Gray-Box (vs. Black-Box) Optimization
- Partition Crossover and Articulation Points
- Deterministic Recombination and Iterated Local Search
- Experiments
- Conclusions and Future Work

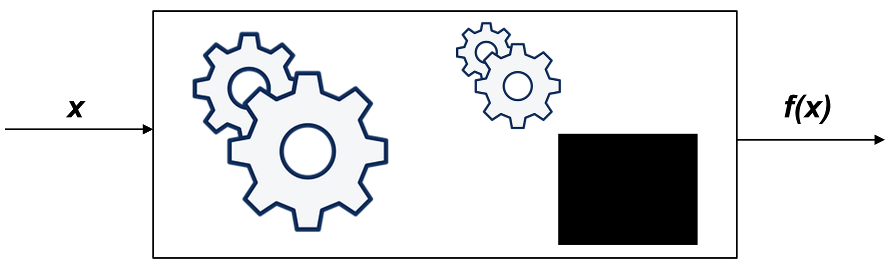
Gray-Box (vs. Black-Box) Optimization



For most of real problems we know (almost) all the details

Gray-Box (vs. Black-Box) Optimization





For most of real problems we know (almost) all the details

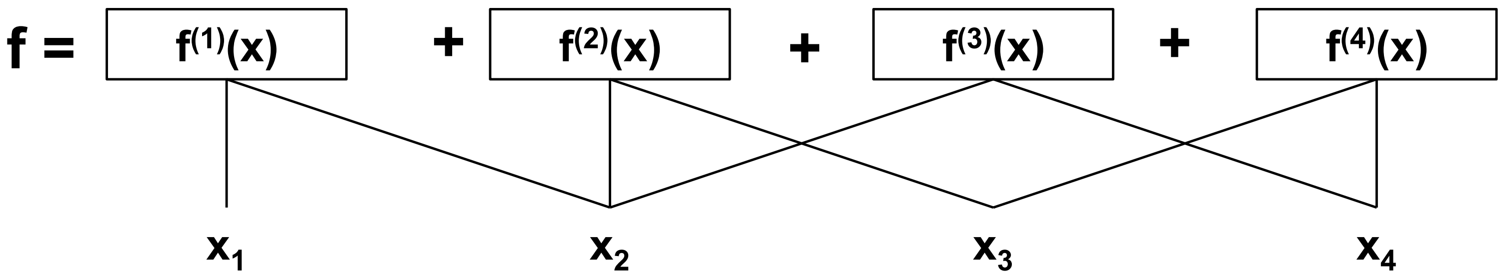
Gray-Box structure: MK Landscapes

$$f(x) = \sum_{i=1}^{m} f^{(i)}(x)$$

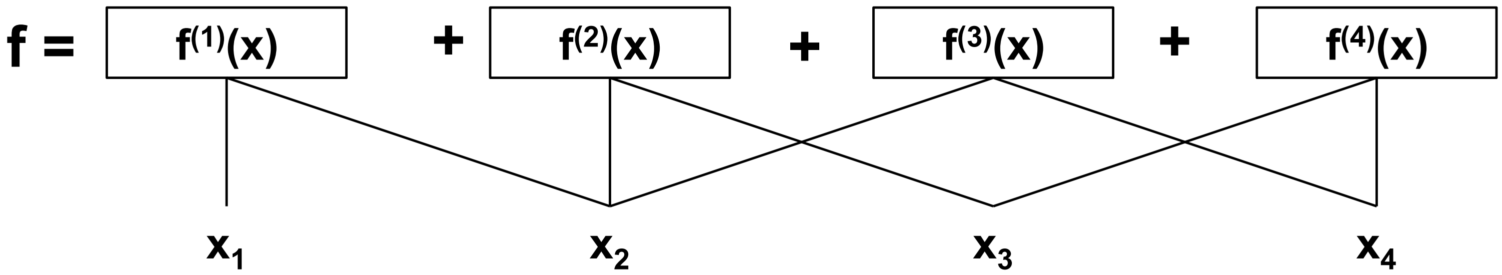
All compresible pseudo-Boolean functions can be transformed into this in polynomial time

Each subfunction is unknown and depends on *k* variables

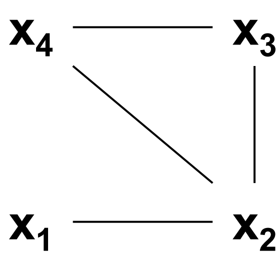
Example (*k*=2):



Variable Interaction



x_i and x_j interact when they appear together in the same subfunction*

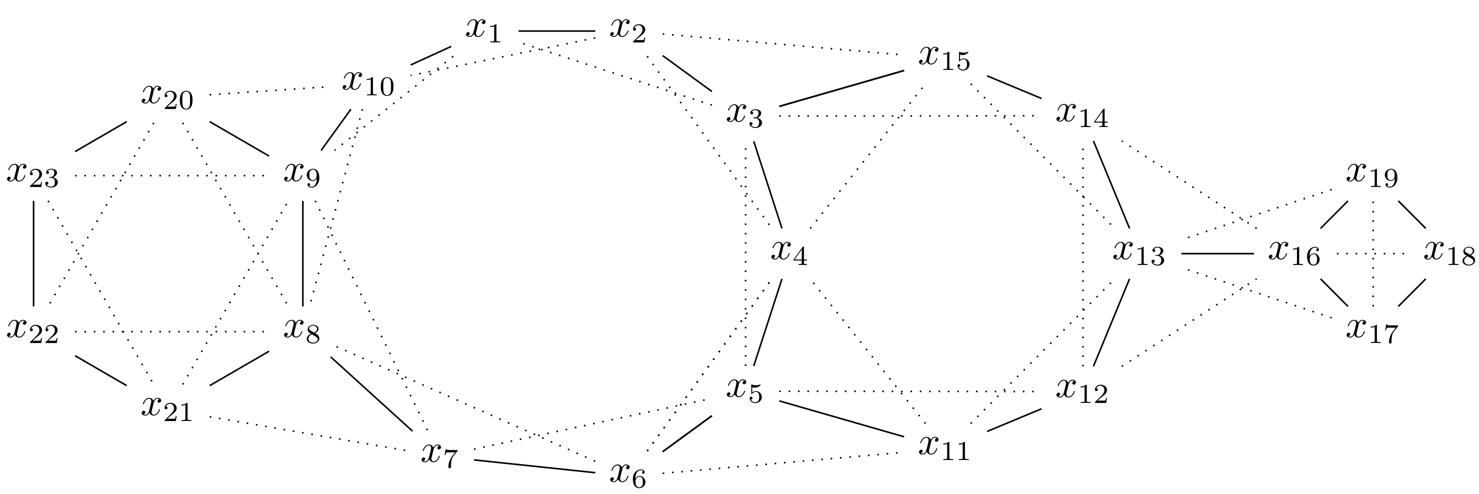


Variable Interaction Graph (VIG)

If x_i and x_j don't interact: $\Delta_{ij} = \Delta_i + \Delta_j$

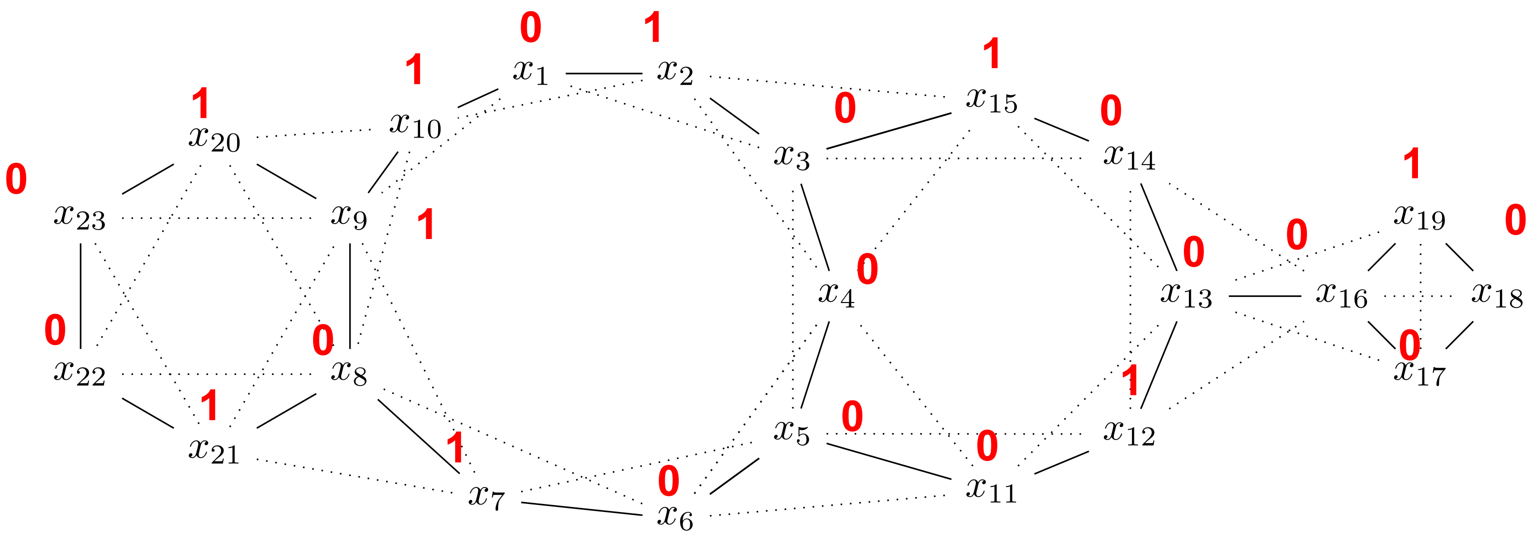
Partition Crossover (PX)

Let us suppose our function has the following VIG…



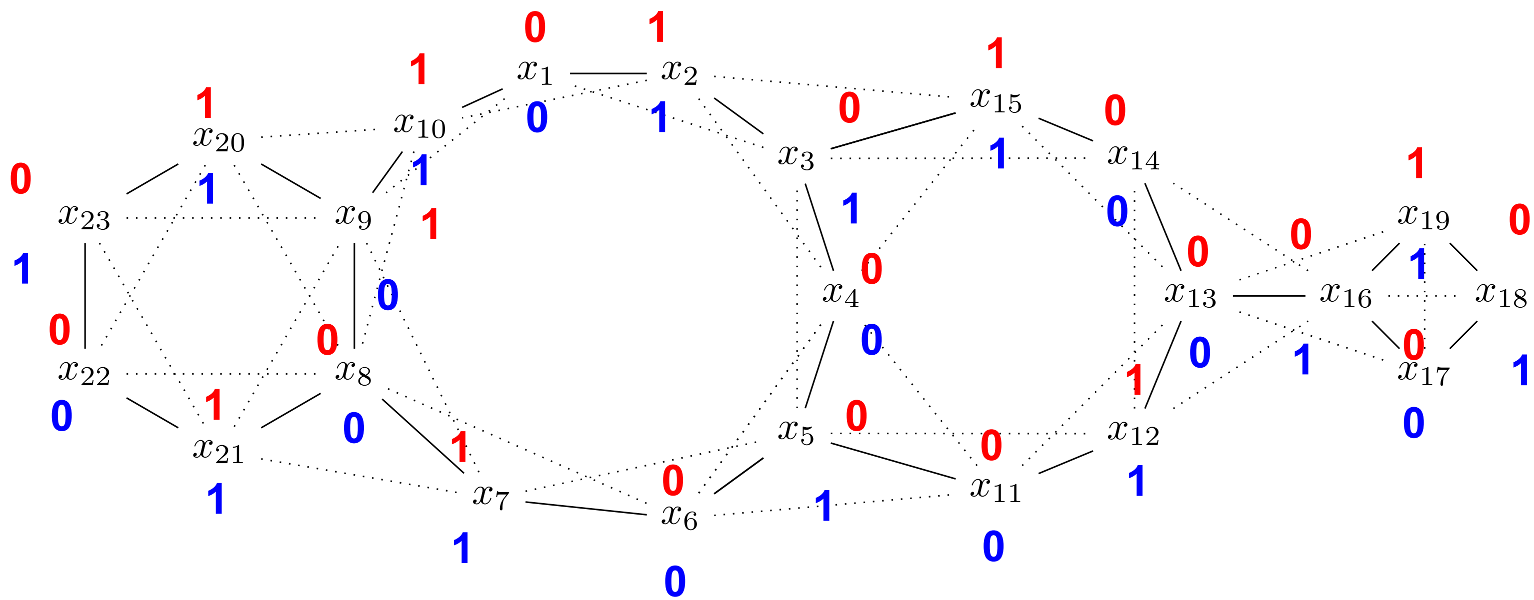
Partition Crossover (PX)

Let us suppose our function has the following VIG…



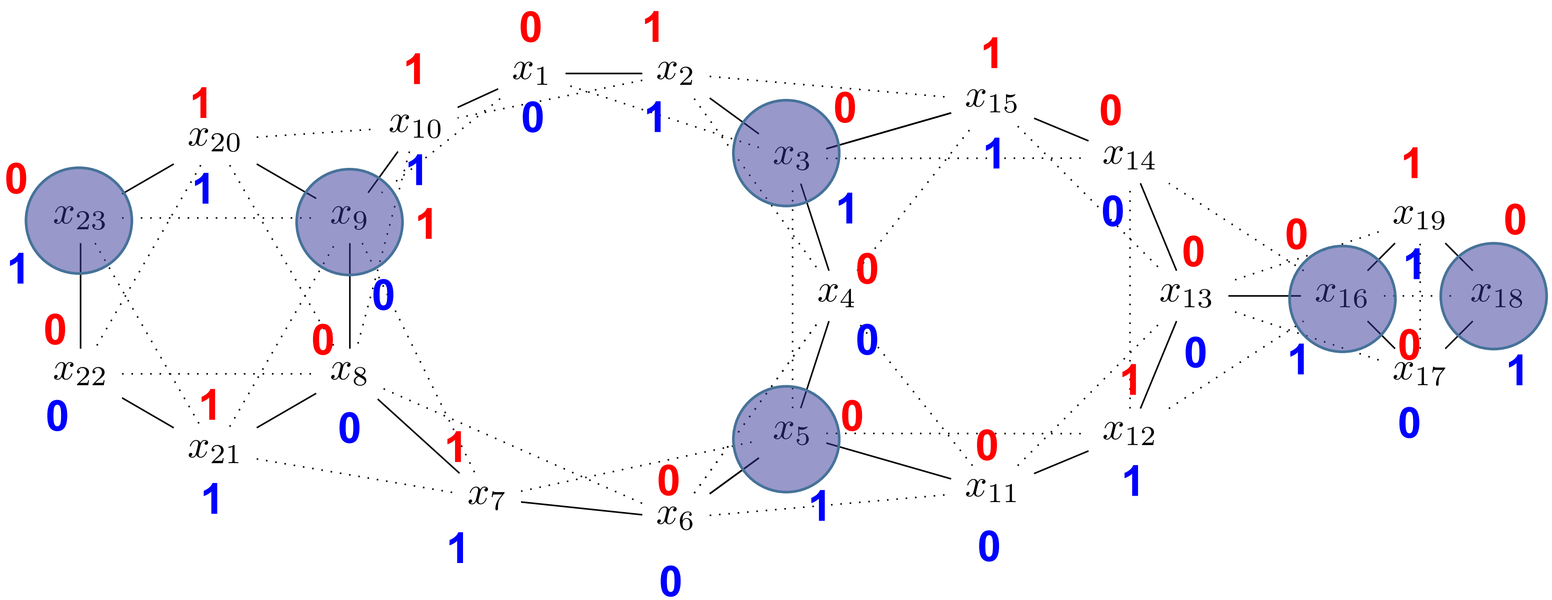
Partition Crossover (PX)

Let us suppose our function has the following VIG…



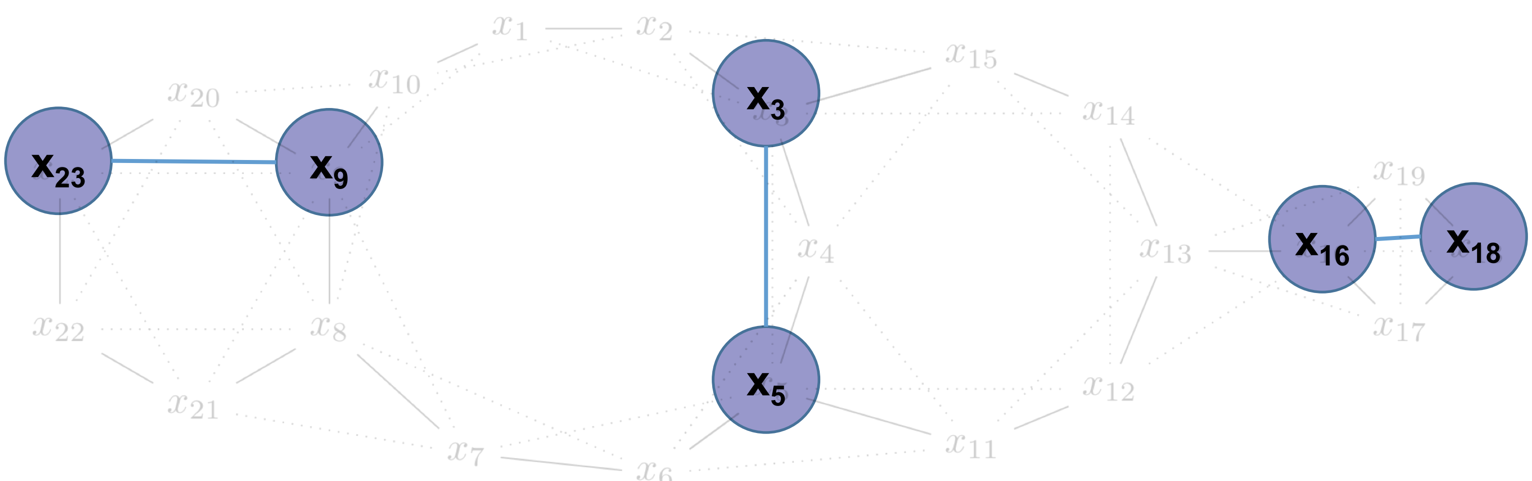
Partition Crossover (PX)

Let us suppose our function has the following VIG…



Partition Crossover (PX)

PX creates a graph with only the differing variables (recombination graph)



All the variables in a component are taken from the same parent

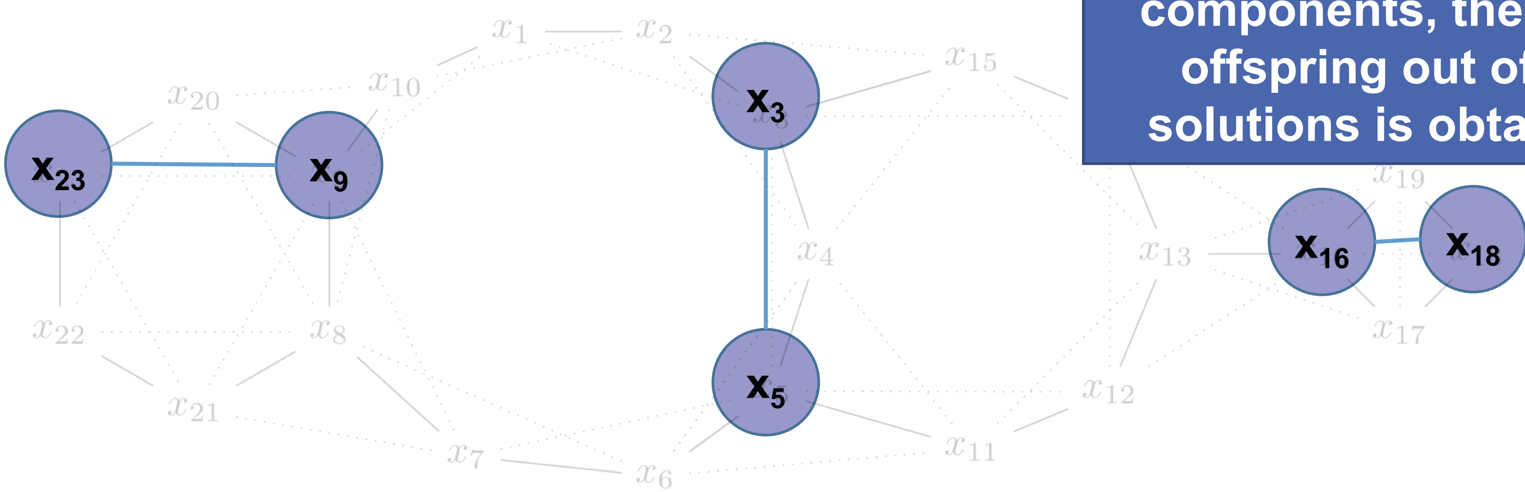
The contribution of each component to the fitness value of the offspring is independent of each other

FOGA 2015: Tinós, Whitley, C.

Partition Crossover (PX)

PX creates a graph with only the differing variables (recombination graph)

If there are q components, the best offspring out of 2^q solutions is obtained

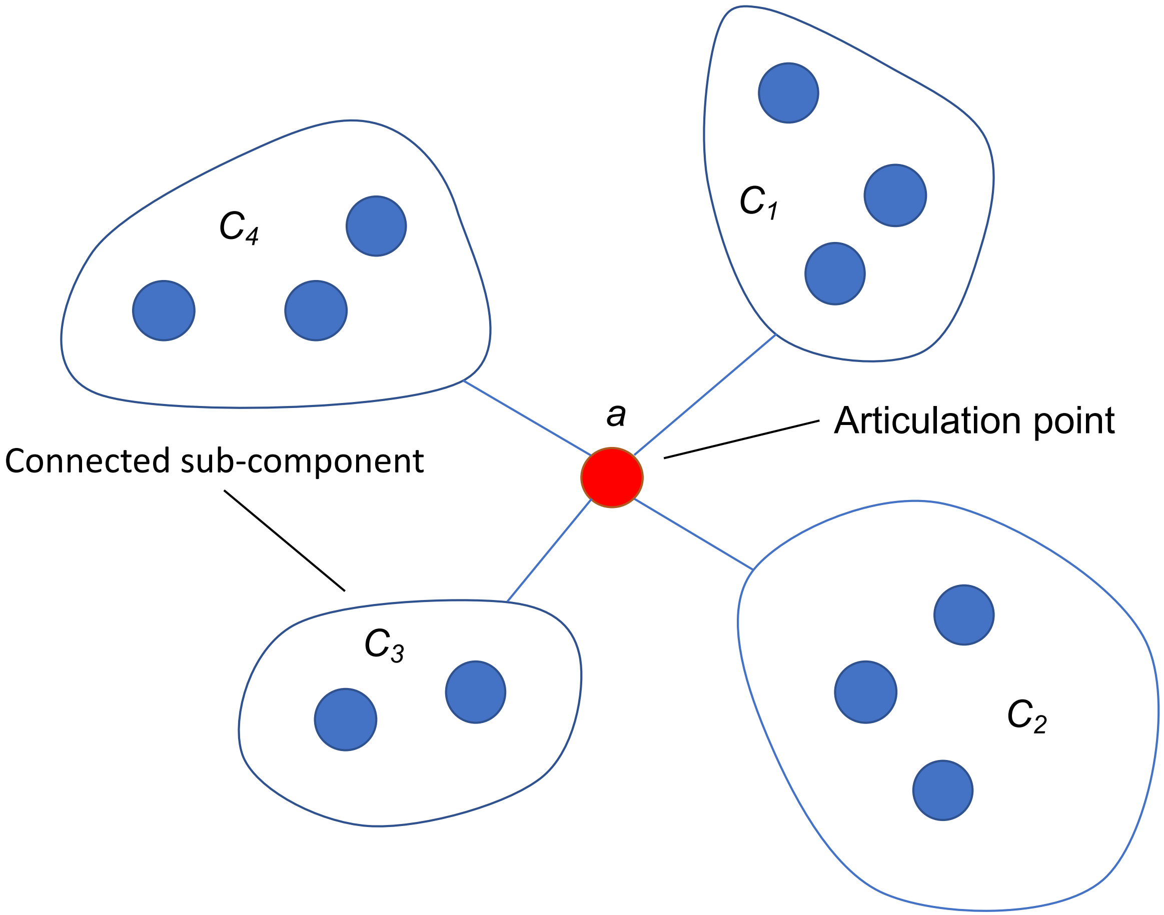


All the variables in a component are taken from the same parent

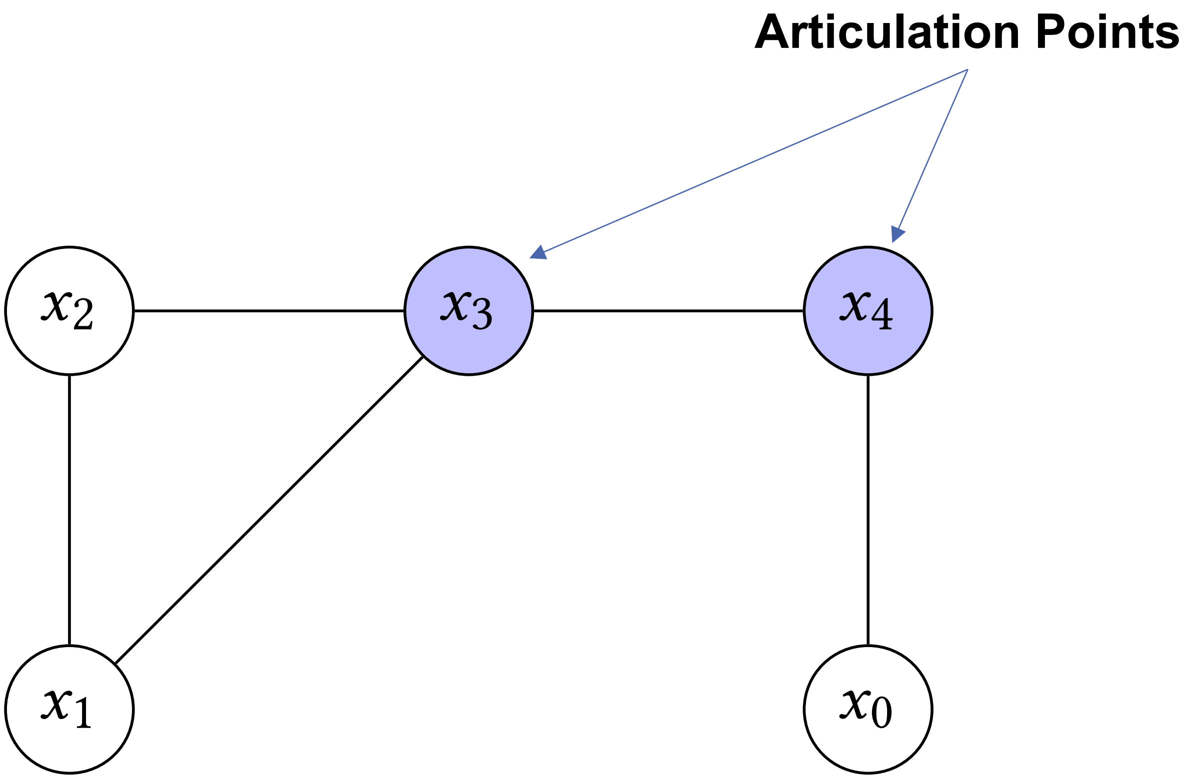
The contribution of each component to the fitness value of the offspring is independent of each other

FOGA 2015: Tinós, Whitley, C.

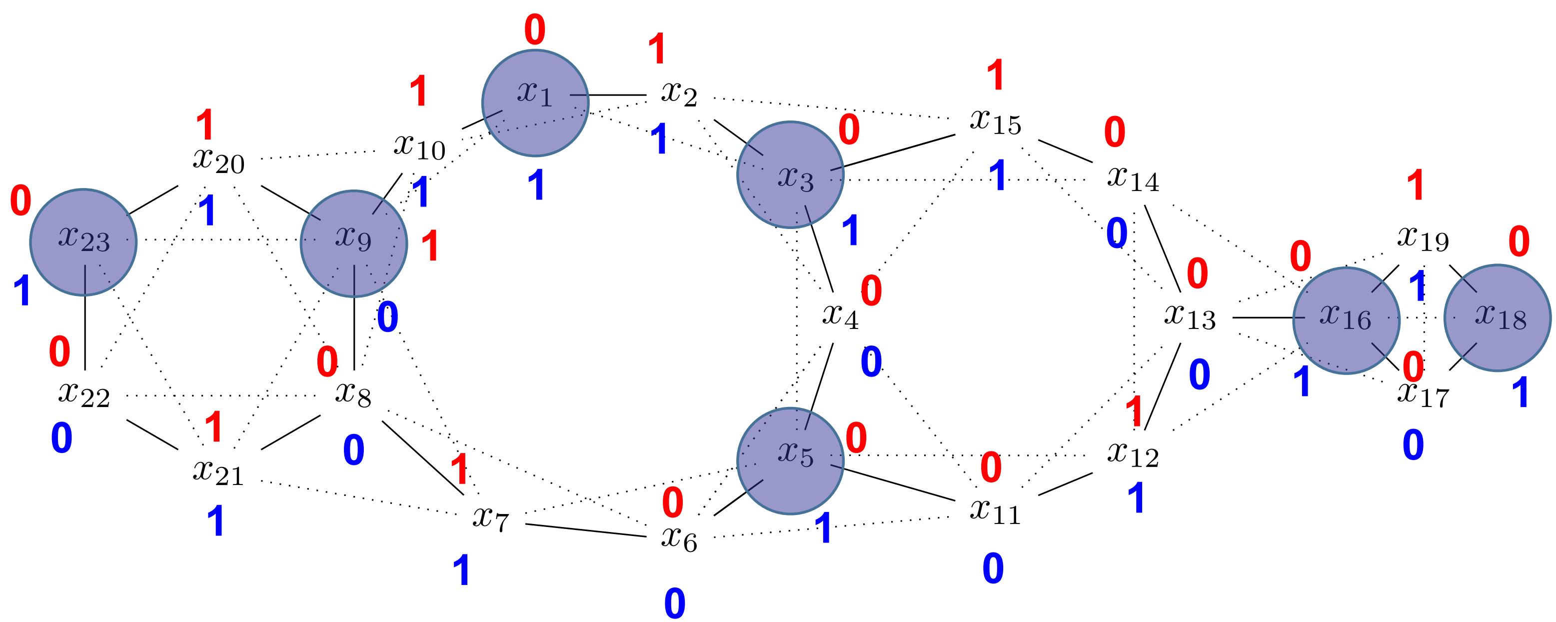
Articulation Points in a Graph



Articulation Points in a Graph

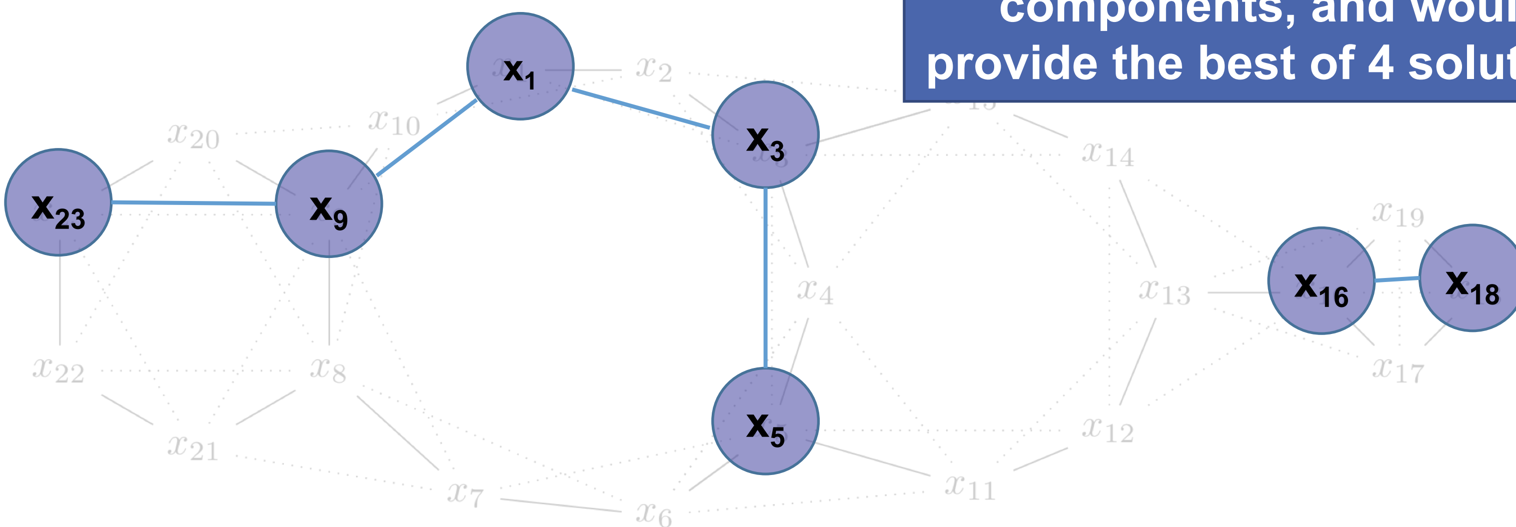


Articulation Points Partition Crossover (APX)



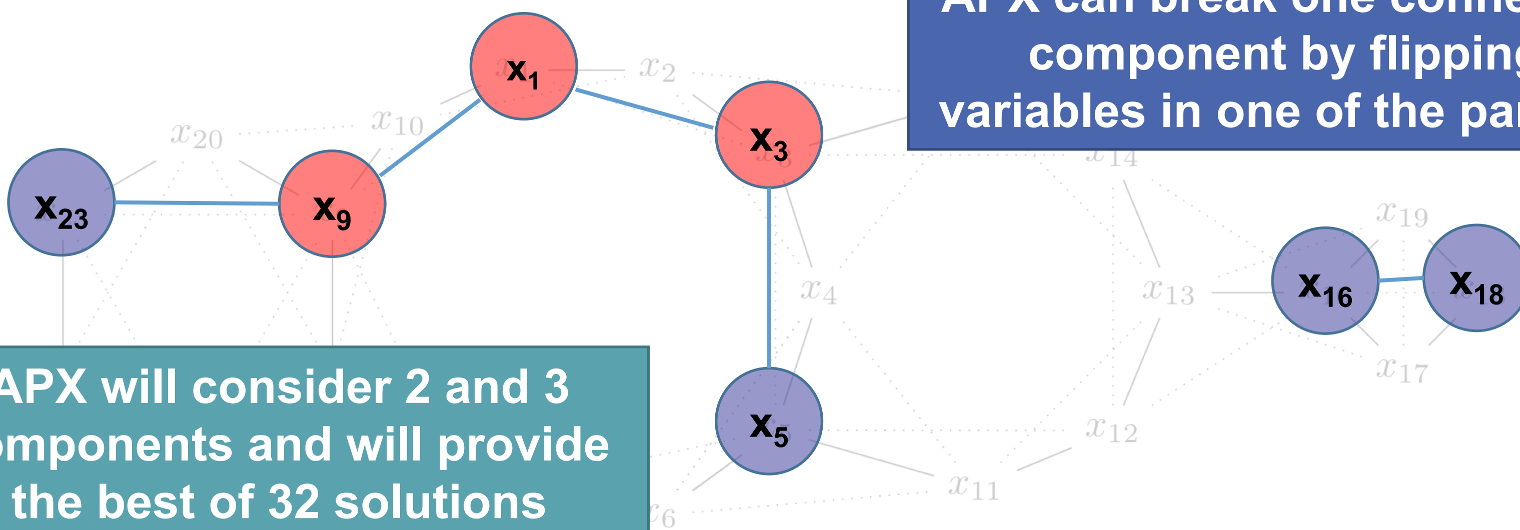
Articulation Points Partition Crossover (APX)

Original PX would find 2 components, and would provide the best of 4 solutions



Articulation Points Partition Crossover (APX)

APX identifies articulation points in the recombination graph



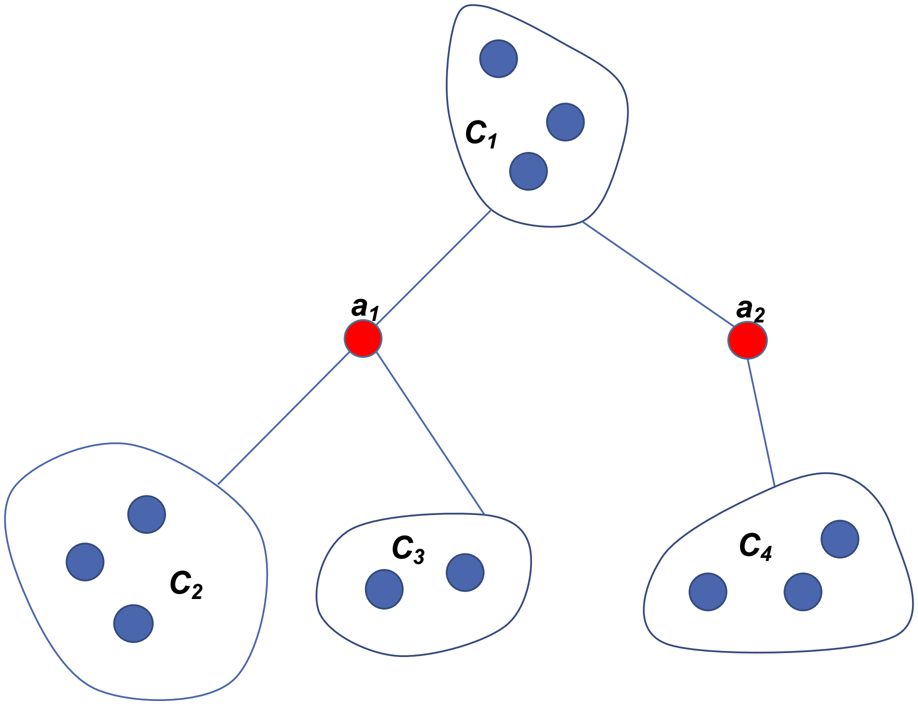
APX can break one connected component by flipping variables in one of the parents

APX will consider 2 and 3 components and will provide the best of 32 solutions

It implicitly considers all the solutions PX would consider if one or none articulation point is removed from each connected component

Articulation Points Partition Crossover (APX)

All the analysis can be done using Tarjan's algorithm to find articulation points (DFS-like algorithm) : time complexity is the same as the original PX



Articulation Points Partition Crossover (APX)

The number of implicitly studied solutions is:

Degree of an articulation point in the recominbation graph

$$E(x,y) = 2^{|CC(G)|} \prod_{C \in CC(G)} \left(1 - e_C + \sum_{a \in AP(C)} \left(2^{d_a} - 1 \right) \right)$$

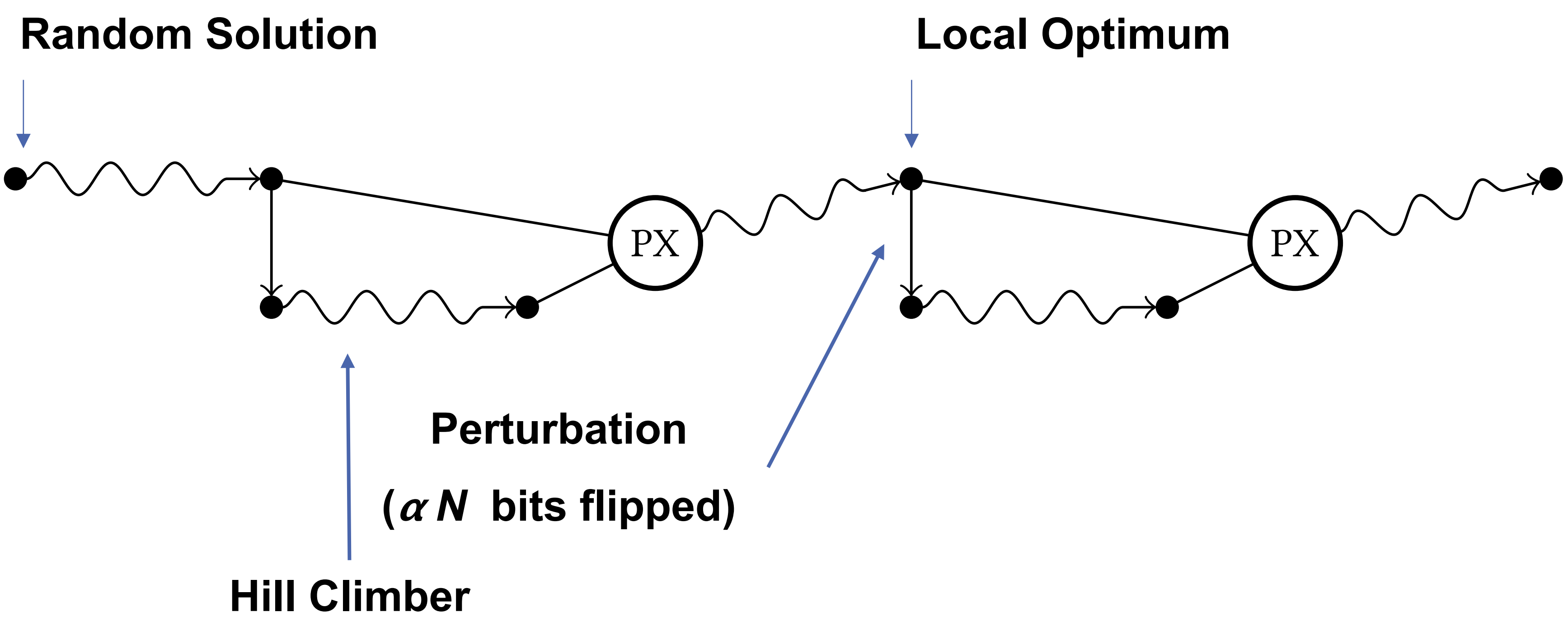
Number of solutions considered by PX

Connected component

Edges joining two articulation points

$$\geq 2^{|CC(G)|} \prod_{\substack{C \in CC(G) \\ |AP(C)|>0}} 2(1 + |AP(C)|)$$

Deterministic Recombination and Iterated Local Search (DRILS)



Experimental Results

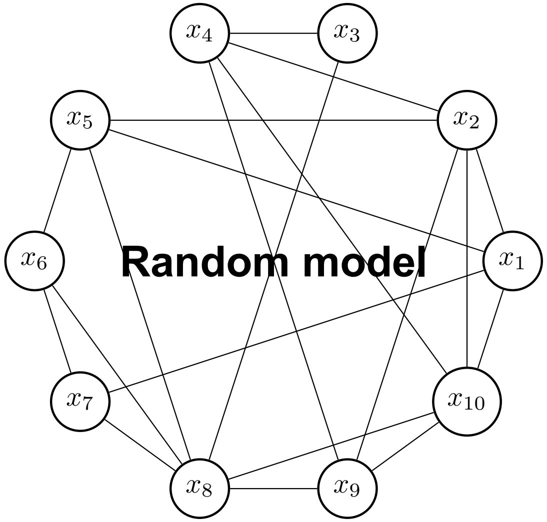
An NK Landscape is a pseudo-Boolean optimization problem with objective function:

$$f(x) = \sum_{l=1}^{N} f^{(l)}(x)$$

where each subfunction $f^{(l)}$ depends on variable x_l and K other variables

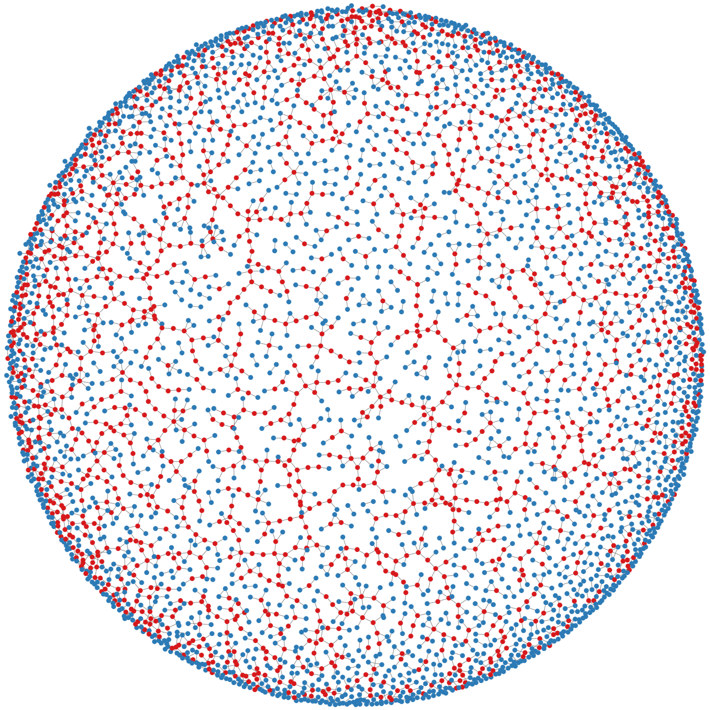
MAX-SAT consists in finding an assignment of variables to Boolean (true and false) values such that the maximum number of clauses is satisfied

A clause is an OR of literals: $x_1 \vee \neg x_2 \vee x_3$



Experimental Results

Example for NKQ Landscapes with N=100 000 and K=2 (DRILS+APX)



There are 4339 nodes grouped in 858 components with 1825 articulation points (in red)

Experimental Results

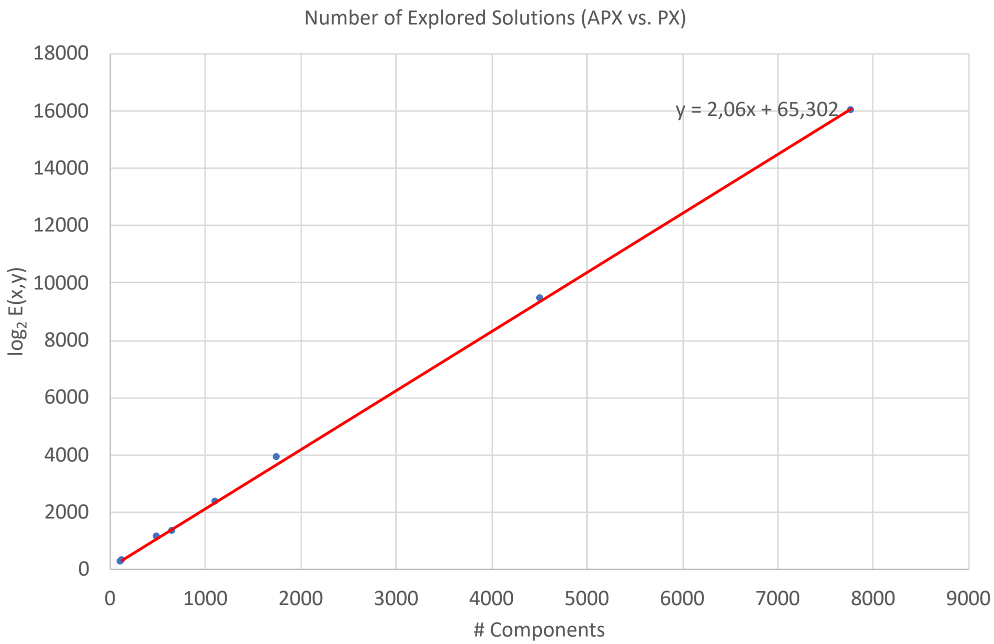
APX runtime is in the same order of magnitude than that of PX

N	K	#Comp.	#APs	d_a	$\log_2 E(x, y)$	Runtime (ms) APX	Runtime (ms) PX
10^5	2	662	687	2.25	1 311	55	46
	3	503	1 151	2.37	1 105	67	73
	4	138	196	2.33	286	55	52
	5	119	218	2.36	254	63	52
10^6	2	7 774	10 836	2.28	15 987	1 383	970
	3	4 515	21 793	2.35	9 454	1 785	2 485
	4	1 748	6 281	2.38	3 907	1 360	1 439
	5	1 105	7 207	2.34	2 341	1 633	1 559

$2^{4515} \approx 10^{1359}$ solutions: $10^{1349} \approx (10^{80})^{16}$ solutions per nanosecond

Experimental Results

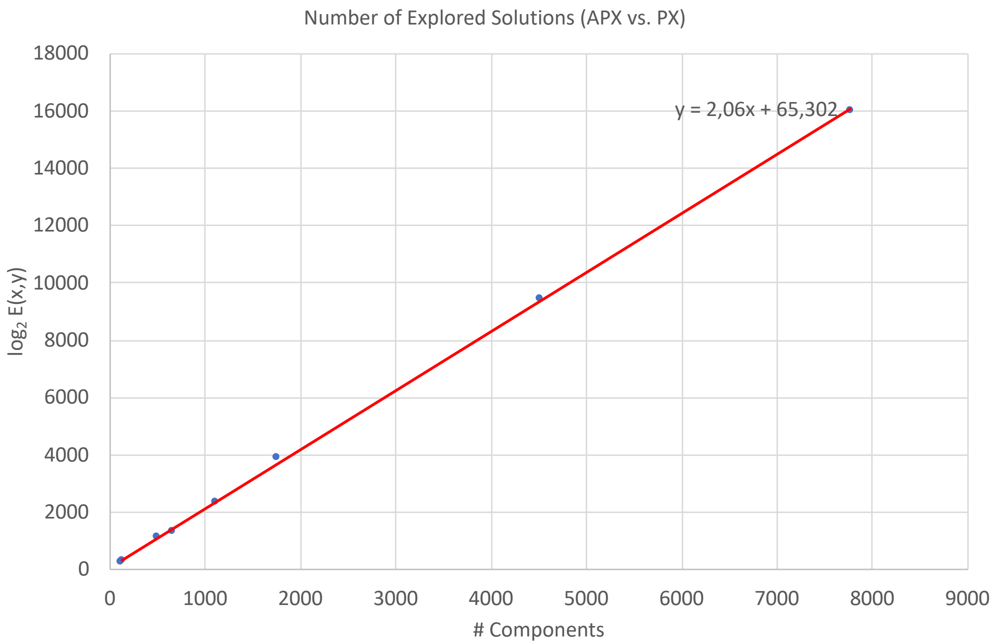
APX runtime is in the same order of magnitude than that of PX

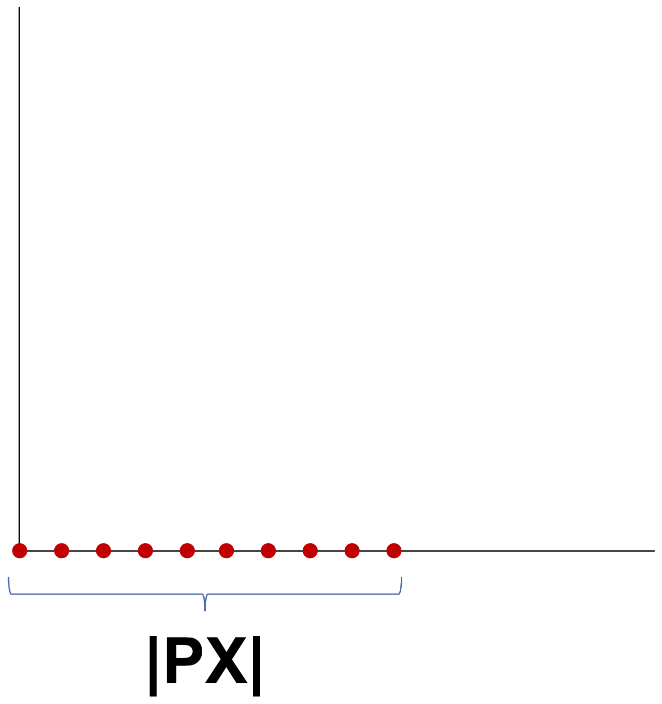


N	K	#Comp.	#APs	d_a	$\log_2 E(x,y)$
10^5	2	662	687	2.25	1 311
	3	503	1 151	2.37	1 105
	4	138	196	2.33	286
	5	119	218	2.36	254
10^6	2	7 774	10 836	2.28	15 987
	3	4 515	21 793	2.35	9 454
	4	1 748	6 281	2.38	3 907
	5	1 105	7 207	2.34	2 341

Experimental Results

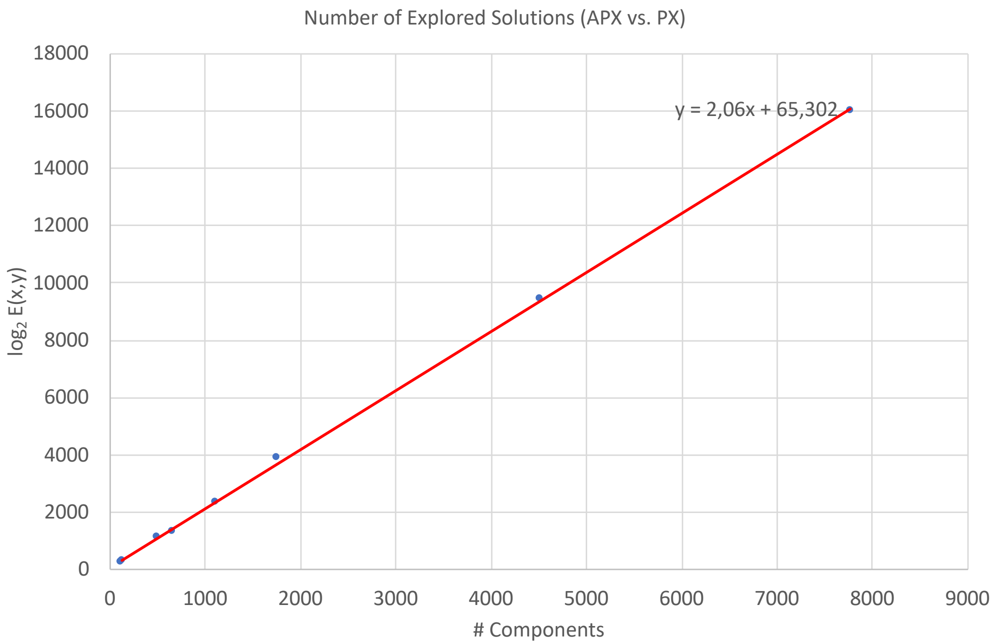
APX runtime is in the same order of magnitude than that of PX

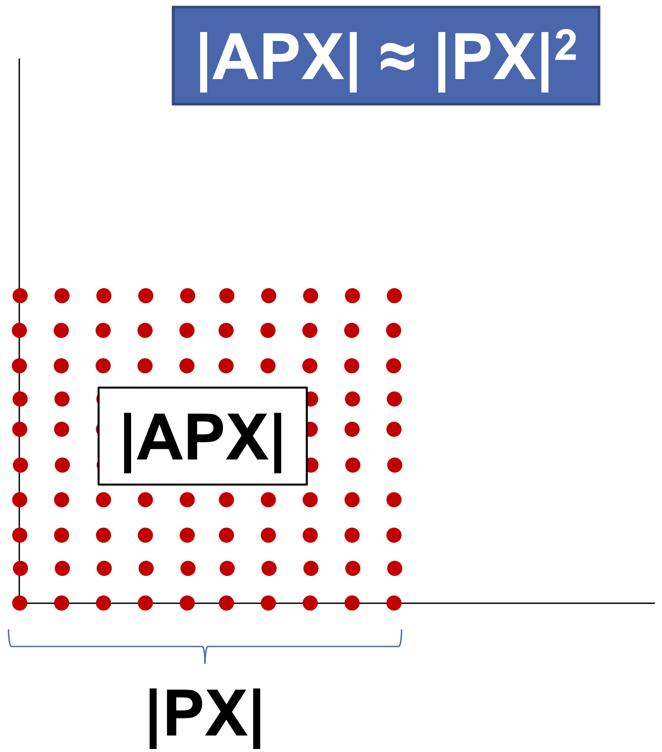




Experimental Results

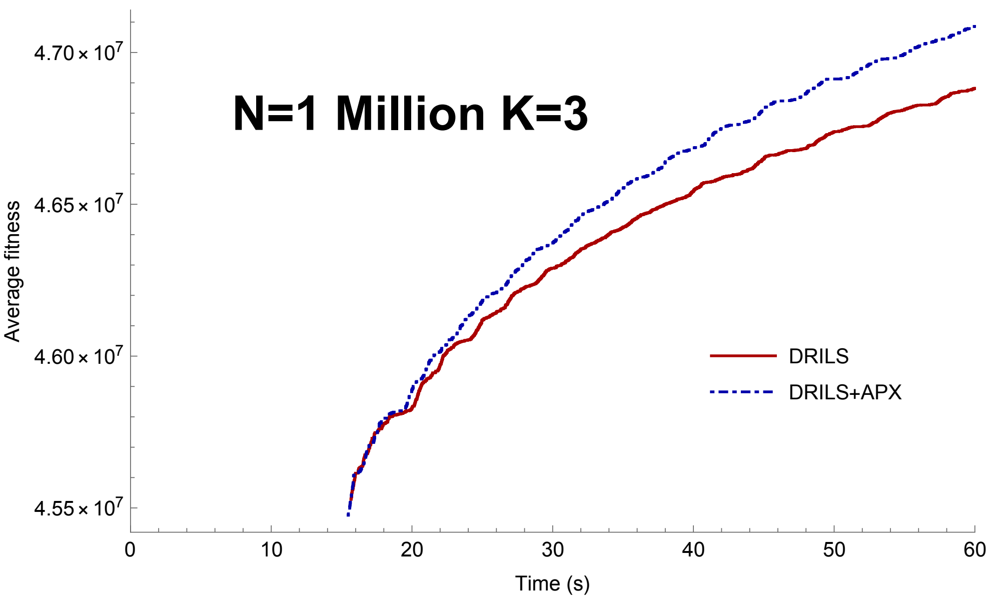
APX runtime is in the same order of magnitude than that of PX and the number of solutions explored is squared!





Experimental Results

DRILS and DRILS+APX solving NKQ Landscapes



		DRILS performance			Runtime (ms)	
N	K	APX	PX	Sim.	APX	PX
10^5	2	10	0	0	55	46
	3	10	0	0	67	73
	4	2	0	8	55	52
	5	1	1	8	63	52
10^6	2	2	3	5	1 383	970
	3	5	0	5	1 785	2 485
	4	9	0	1	1 360	1 439
	5	1	0	9	1 633	1 559

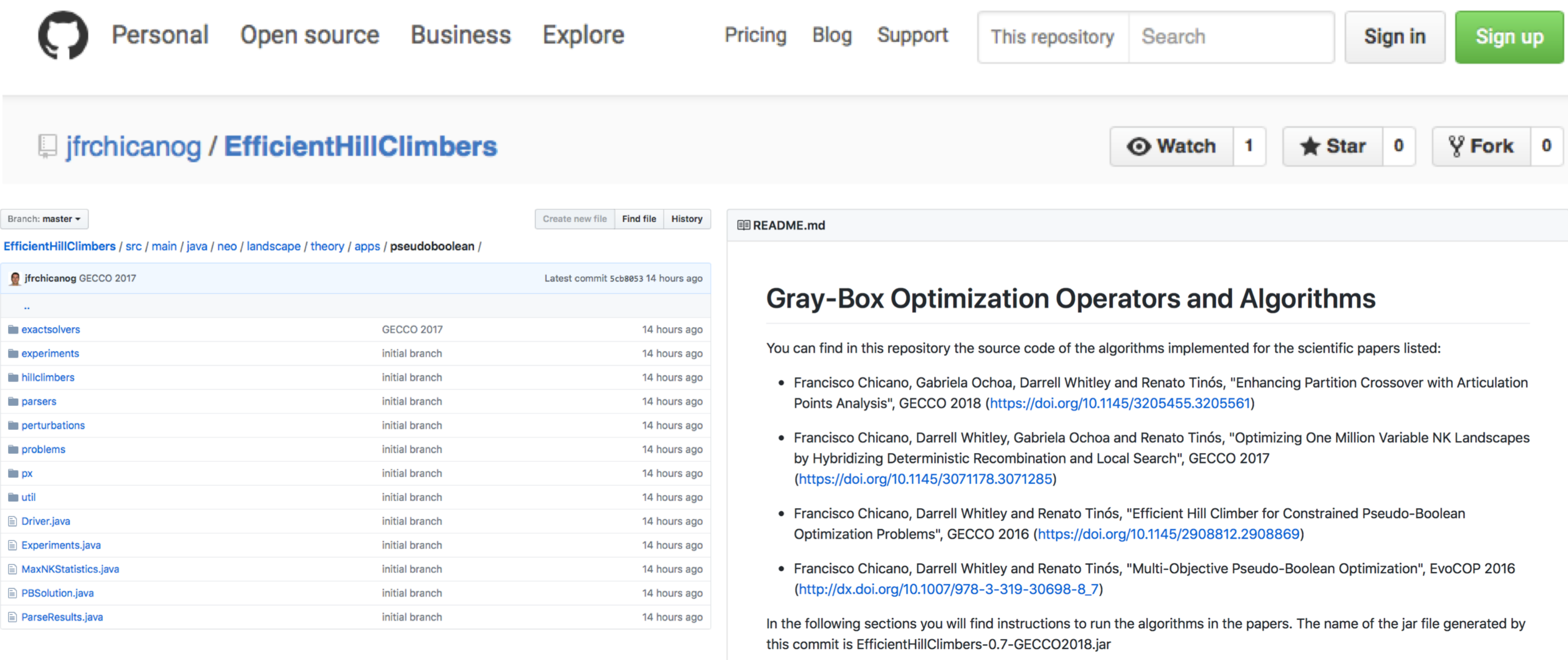
Experimental Results

DRILS and DRILS+APX solving MAX-SAT (instances from MAX-SAT Evaluation 2017)

		DRILS performance			Runtime (μs)	
Instances	α	APX	PX	Sim.	APX	PX
Unweighted	0.10	78	1	81	463	454
	0.20	82	2	75	684	729
	0.30	85	2	73	849	1 060
Weighted	0.10	26	19	87	1 425	882
	0.20	49	14	69	1 859	1 416
	0.30	77	5	50	2 365	1 713

Source Code in GitHub

https://github.com/jfrchicanog/EfficientHillClimbers



Conclusions

- The Variable Interaction Graph provides useful information to improve the search
- Articulation Points Partition Crossover squares the number of solutions considered by PX in around the same time
- APX is specially good in Unweighted MAX-SAT instances (many plateaus)
- Take home message: use Gray-Box Optimization if you can

Future Work

- Plateaus exploration in MAX-SAT guided by APX
- New ways of perturbing the solution to maximize the components in (A)PX
- Look at the Variable Interaction Graph of industrial problems

GECCO

Acknowledgements

GOBIERNO DE ESPAÑA
MINISTERIO DE EDUCACIÓN, CULTURA Y DEPORTE

GOBIERNO DE ESPAÑA
MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD

FULBRIGHT España

CNPq

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
UNITED STATES AIR FORCE

UNIVERSITAS MALACITANA
UNIVERSIDAD DE MÁLAGA

ANDALUCÍA TECH
Campus de Excelencia Internacional

Colorado State University

FAPESP

The Leverhulme Trust

Thanks for your attention!!!