CIMO 2018

Gray-Box Optimization for Million Variable Pseudo-Boolean Problems

Francisco Chicano

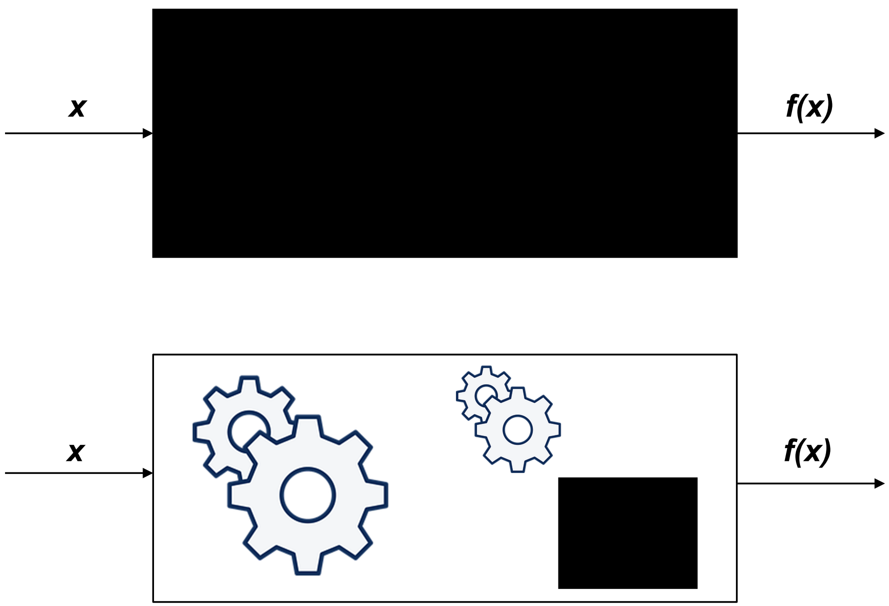
Joint work with

Darrell Whitley, Renato Tinós, Gabriela Ochoa and Andrew M. Sutton

Outline

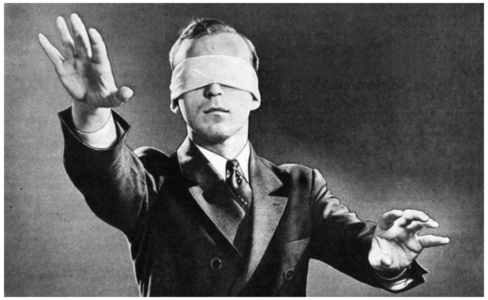
- **Gray-Box (vs. Black-Box) Optimization**
- **Hamming Ball Hill Climber and Partition Crossover**
- **Deterministic Recombination and Iterated Local Search**
- **Experiments**
- **Conclusions and Future Work**

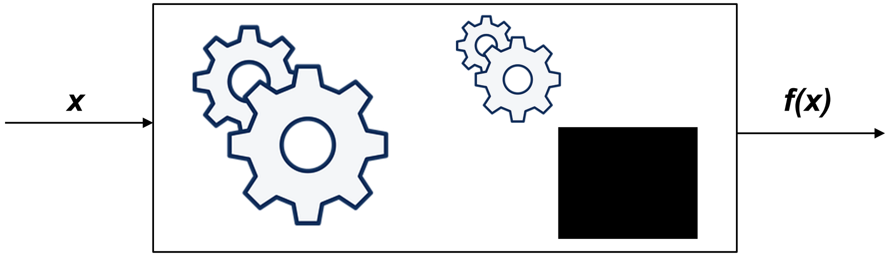
Gray-Box (vs. Black-Box) Optimization



For most of real problems we know (almost) all the details

Gray-Box (vs. Black-Box) Optimization





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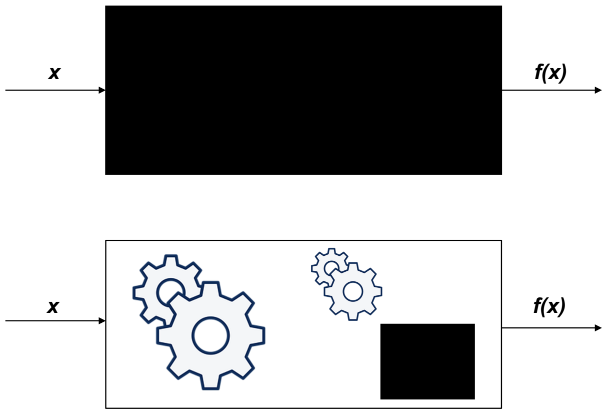
Gray-Box (vs. Black-Box) Optimization



OneMax
$\Theta(n/\log n)$
$\Theta(1)$

Other $\Theta(1)$-solvable problems:

- **Leading Ones**
- **Trap Functions**
- **Jump Functions**
- **Massively Multimodal Deceptive Problem**

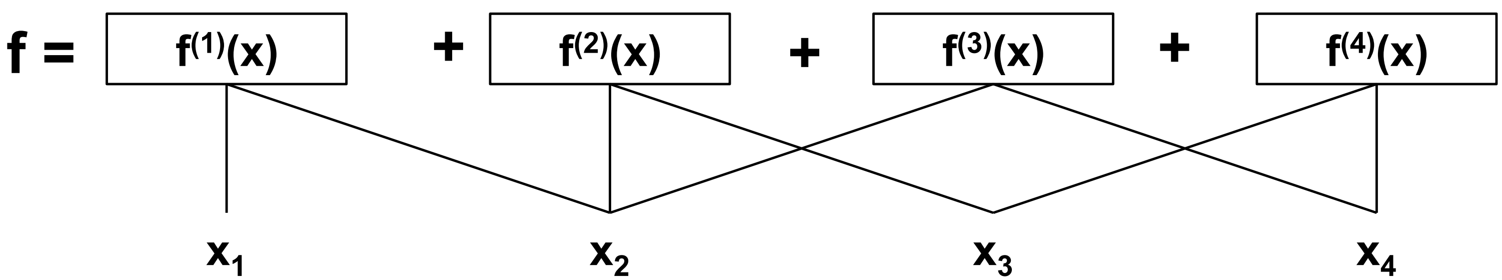
Gray-Box structure: MK Landscapes

$$f(x) = \sum_{i=1}^{m} f^{(i)}(x)$$

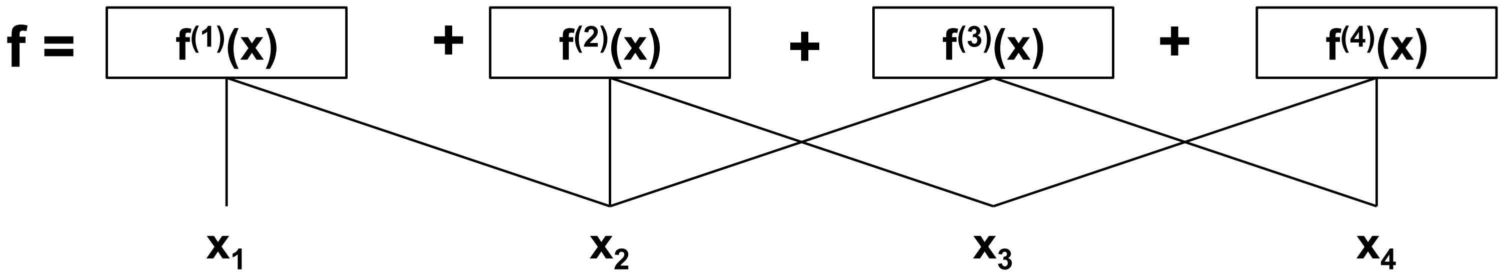
All compresible pseudo-Boolean functions can be transformed into this in polynomial time

Each subfunction is unknown and depends on *k* variables

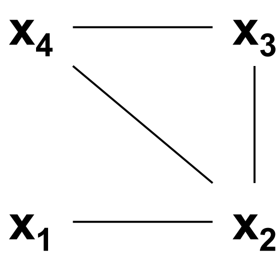
Example (*k*=2)*:*



Variable Interaction



x_i and x_j interact when they appear together in the same subfunction*

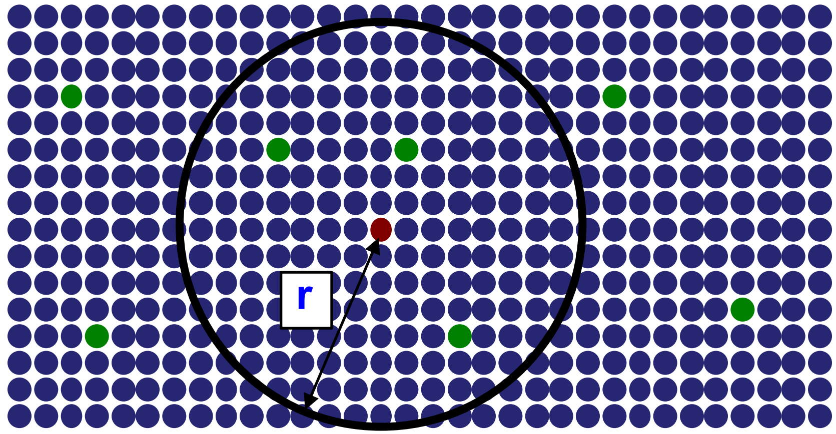


Variable Interaction Graph (VIG)

If x_i and x_j don't interact: $\Delta_{ij} = \Delta_i + \Delta_j$

Hamming Ball Hill Climber (HBHC)

Identifying improving moves in a ball of radius *r* around solution *x*

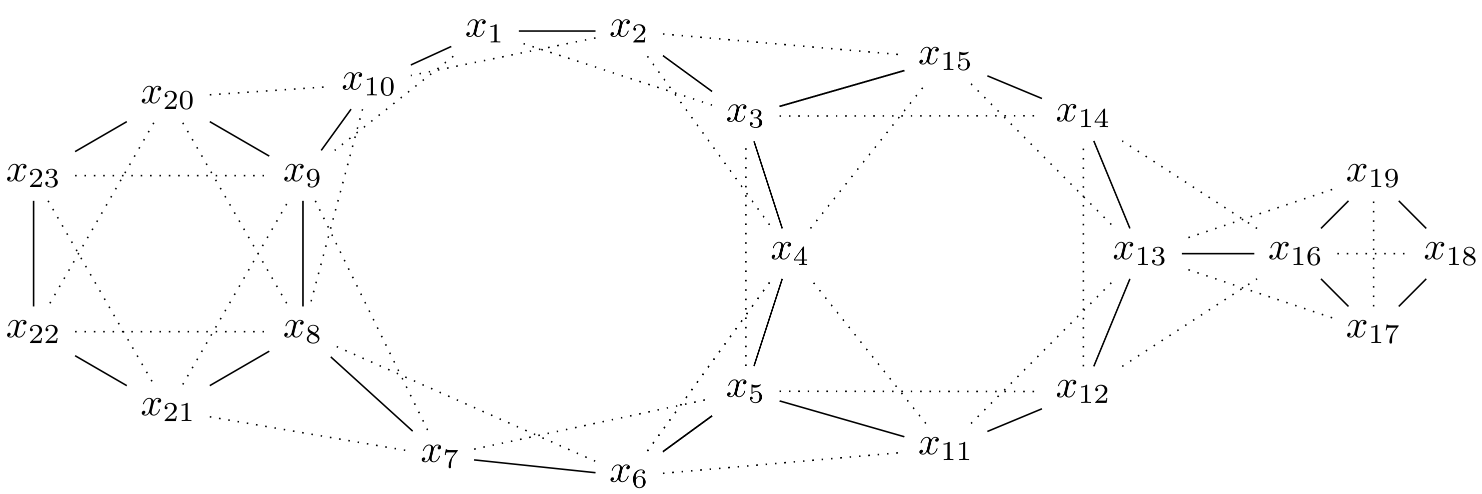


Based on the concept of Score (delta evaluation): Δ_v

GECCO 2014: C., Whitley, Sutton

Partition Crossover (PX)

Let us suppose our function has the following VIG…



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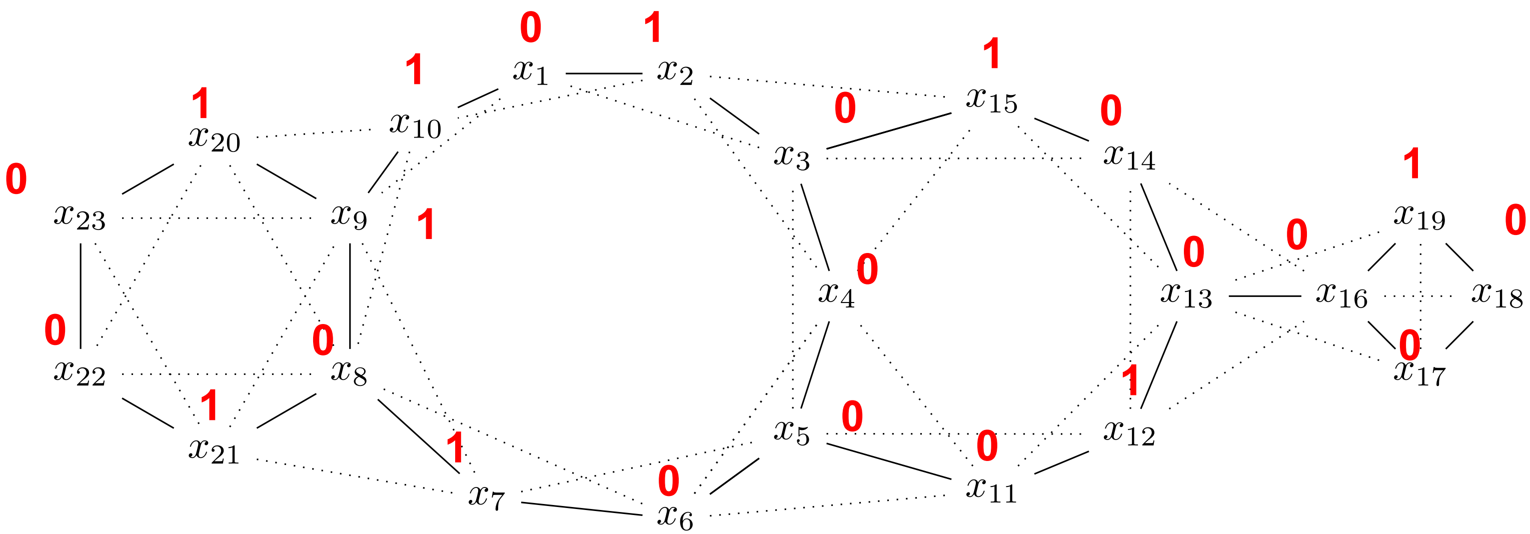

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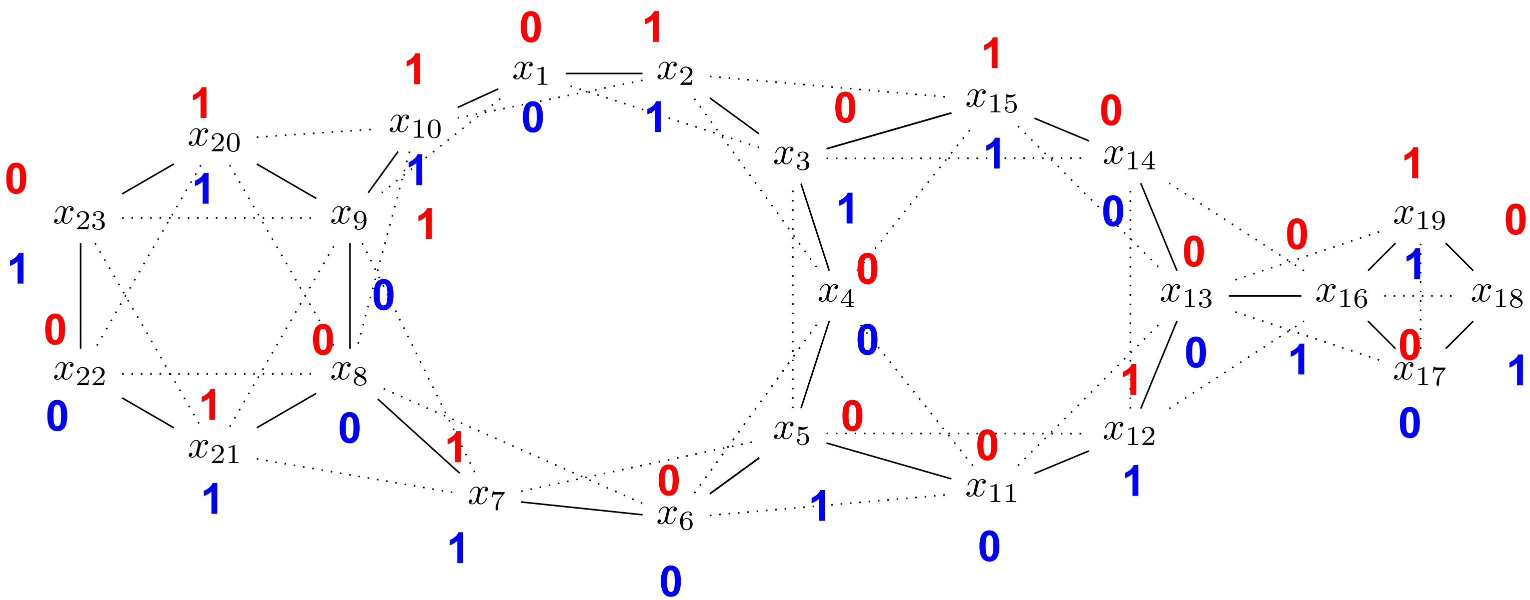

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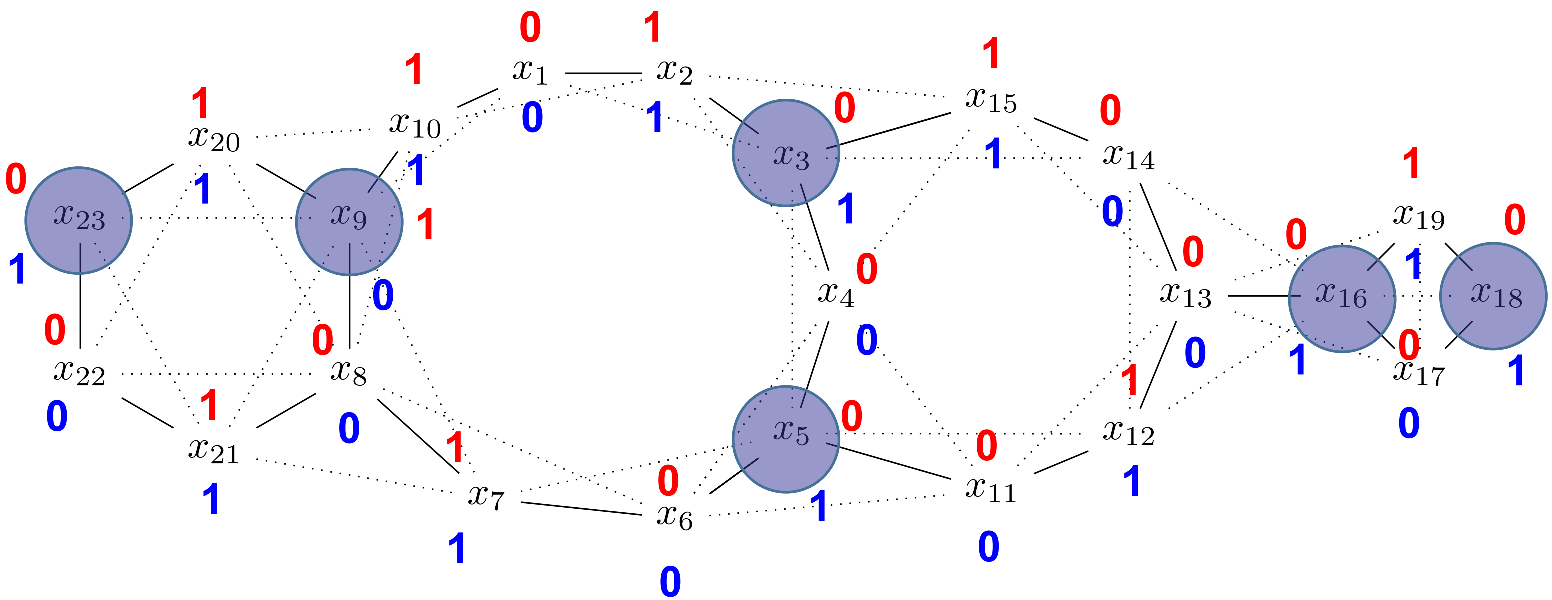
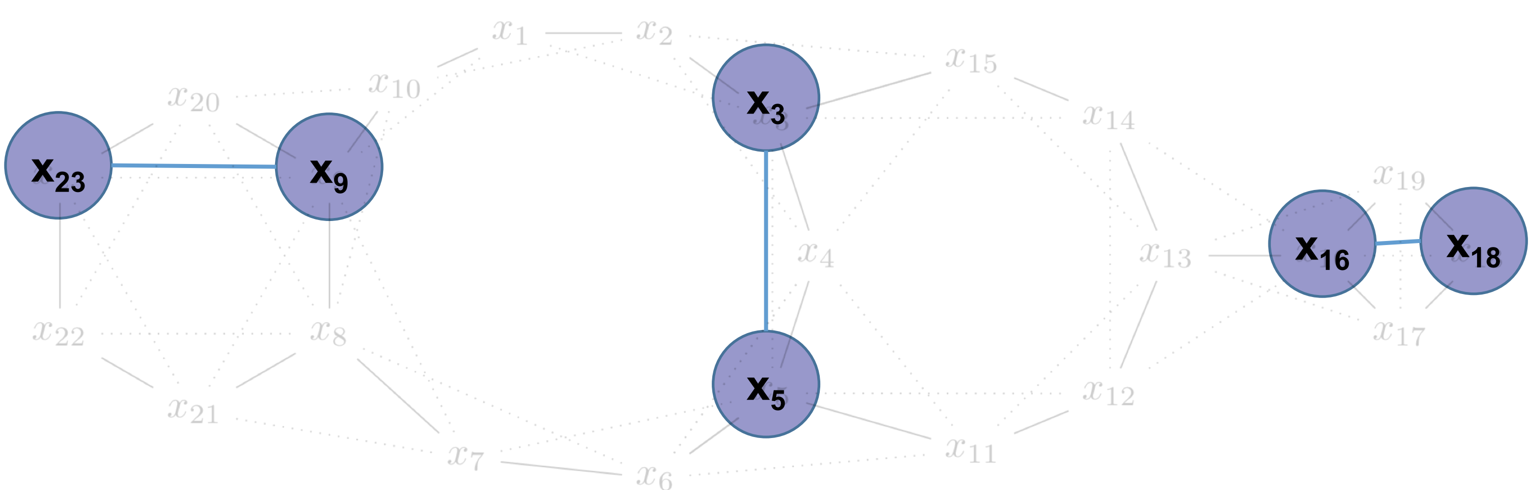


Partition Crossover (PX)

PX creates a graph with only the differing variables (recombination graph)



All the variables in a component are taken from the same parent

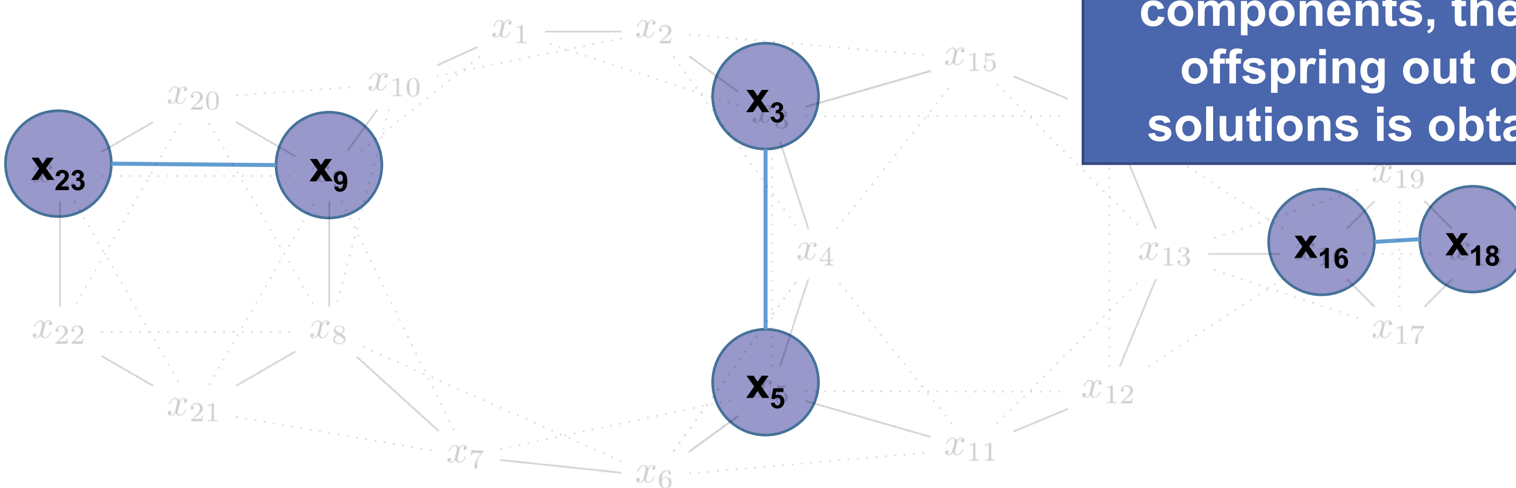
The contribution of each component to the fitness value of the offspring is independent of each other

FOGA 2015: Tinós, Whitley, C.

Partition Crossover (PX)

PX creates a graph with only the differing variables (recombination graph)

If there are q components, the best offspring out of 2^q solutions is obtained



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Articulation Points Partition Crossover (APX)

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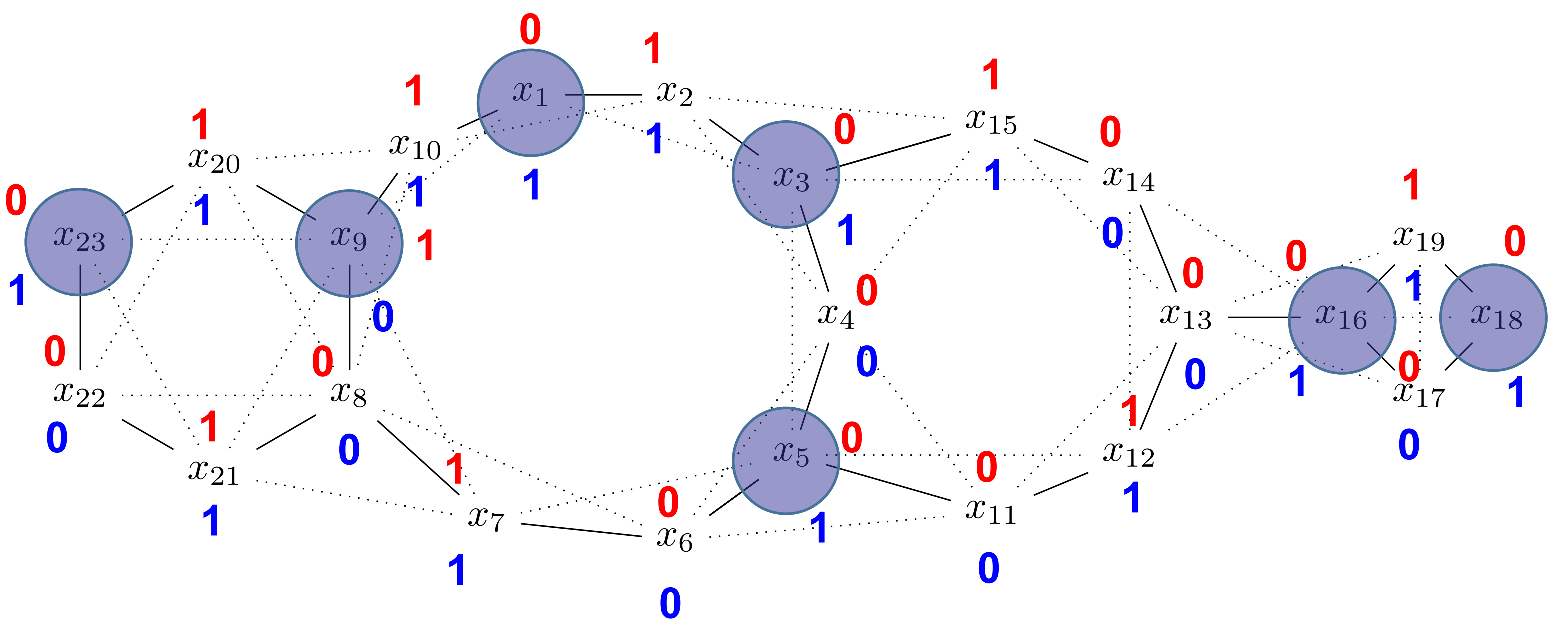
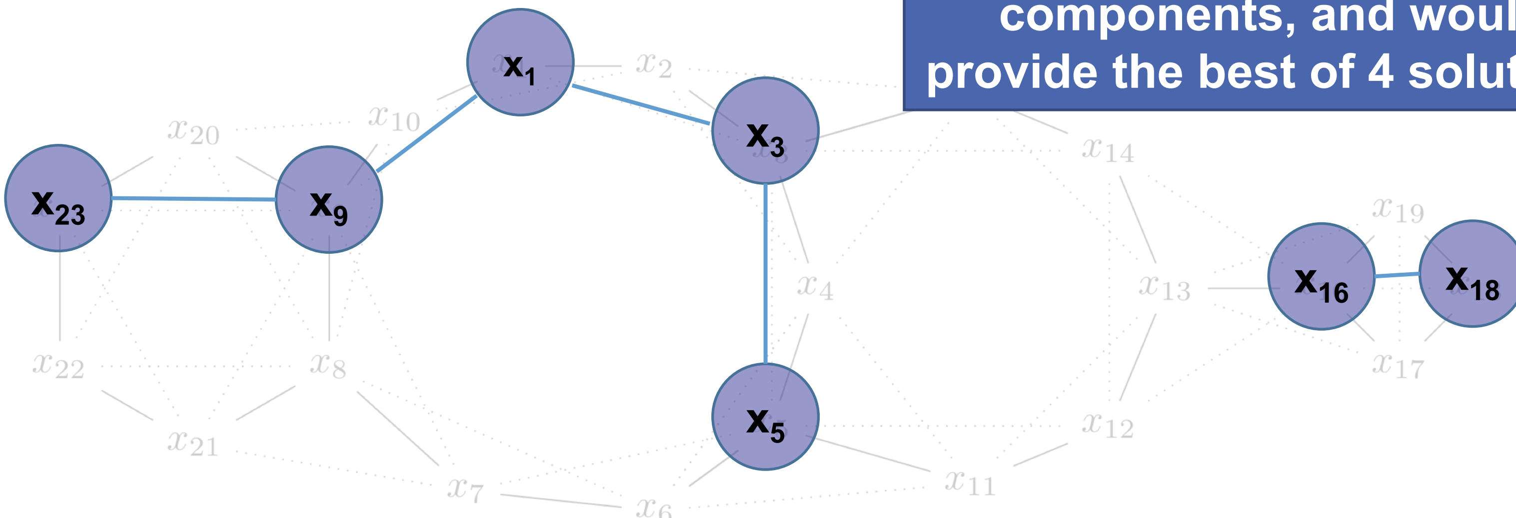


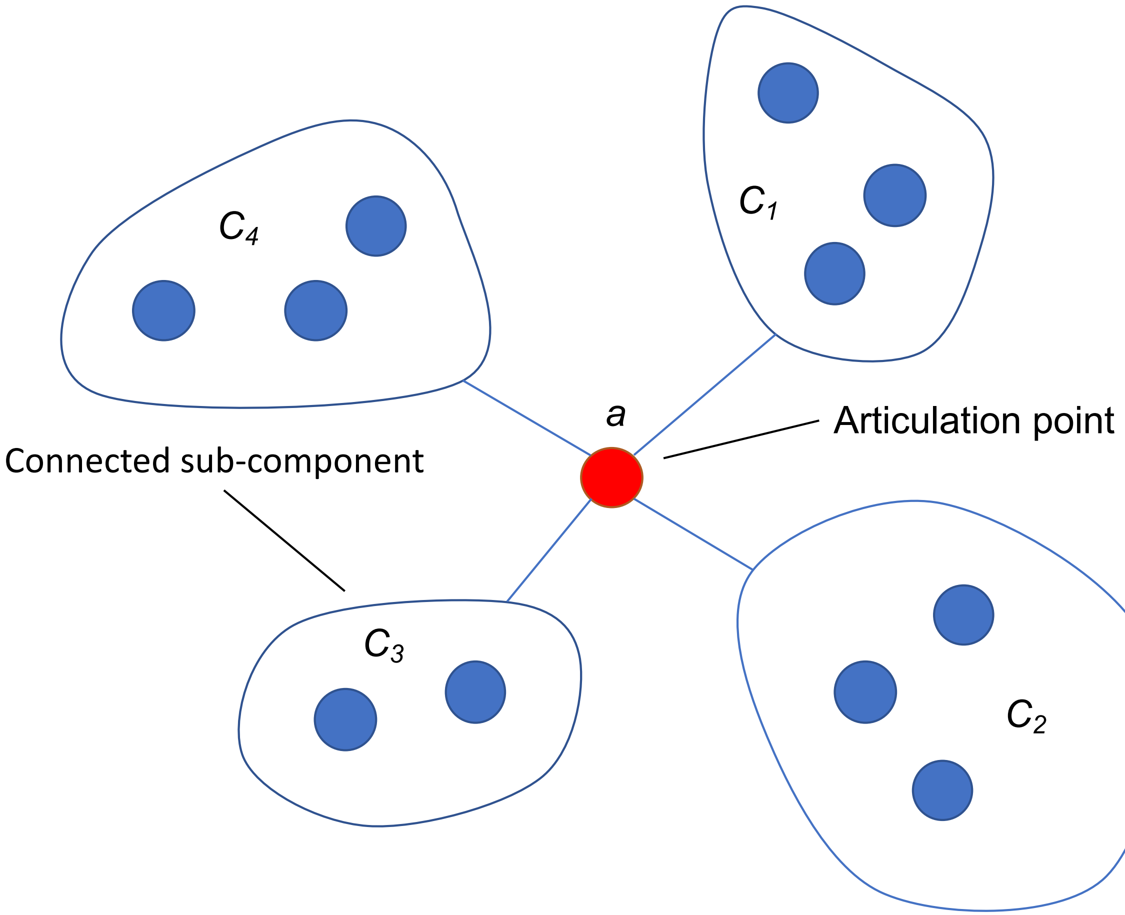
Articulation Points Partition Crossover (APX)

Original PX would find 2 components, and would provide the best of 4 solutions

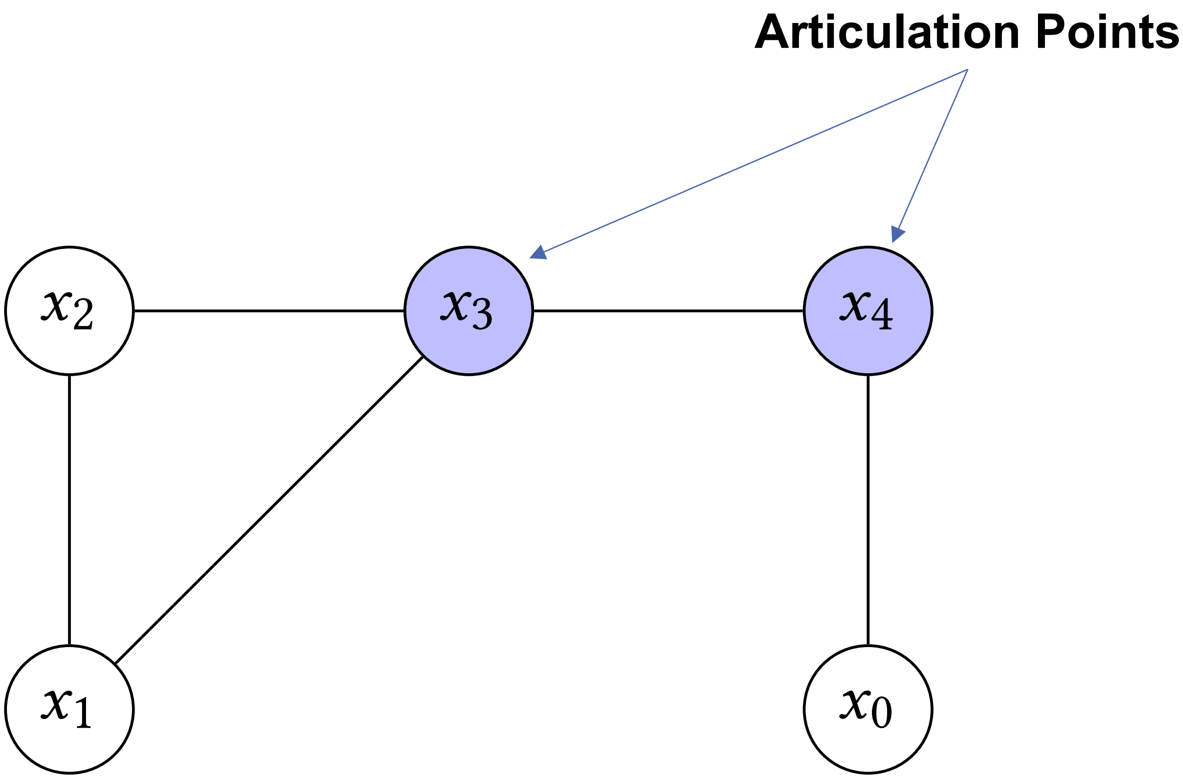


GECCO 2018: C., Ochoa, Whitley, Tinós

Articulation Points in a Graph

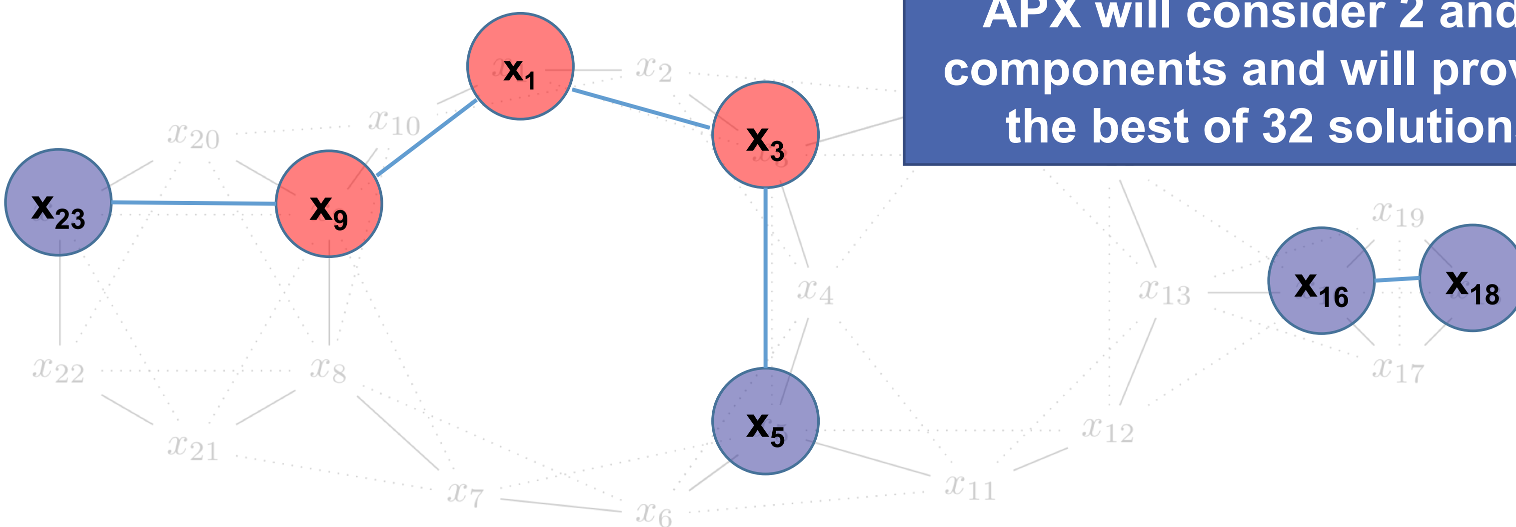


Articulation Points in a Graph



Articulation Points Partition Crossover (APX)

APX identifies articulation points in the recombination graph



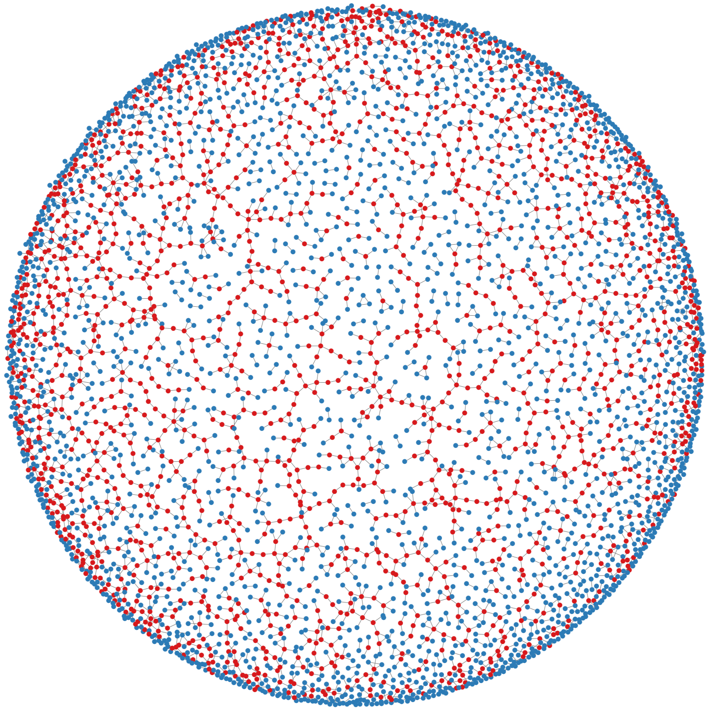
APX will consider 2 and 3 components and will provide the best of 32 solutions

It implicitly considers all the solutions PX would consider if one or none articulation point is removed from each connected component

GECCO 2018: C., Ochoa, Whitley, Tinós

Articulation Points Partition Crossover (APX)

Example for NKQ Landscapes with N=100 000 and K=2 (DRILS+APX)



There are 4339 nodes grouped in 858 components with 1825 articulation points (in red)

GECCO 2018: C., Ochoa, Whitley, Tinós

Articulation Points Partition Crossover (APX)

The number of implicitly studied solutions is:

$$E(x,y) = 2^{|CC(G)|} \prod_{C \in CC(G)} \left(1 - e_C + \sum_{a \in AP(C)} \left(2^{d_a} - 1 \right) \right)$$

Degree of an articulation point in the recominbation graph (d_a)

Number of solutions considered by PX ($2^{|CC(G)|}$)

Connected component ($C \in CC(G)$)

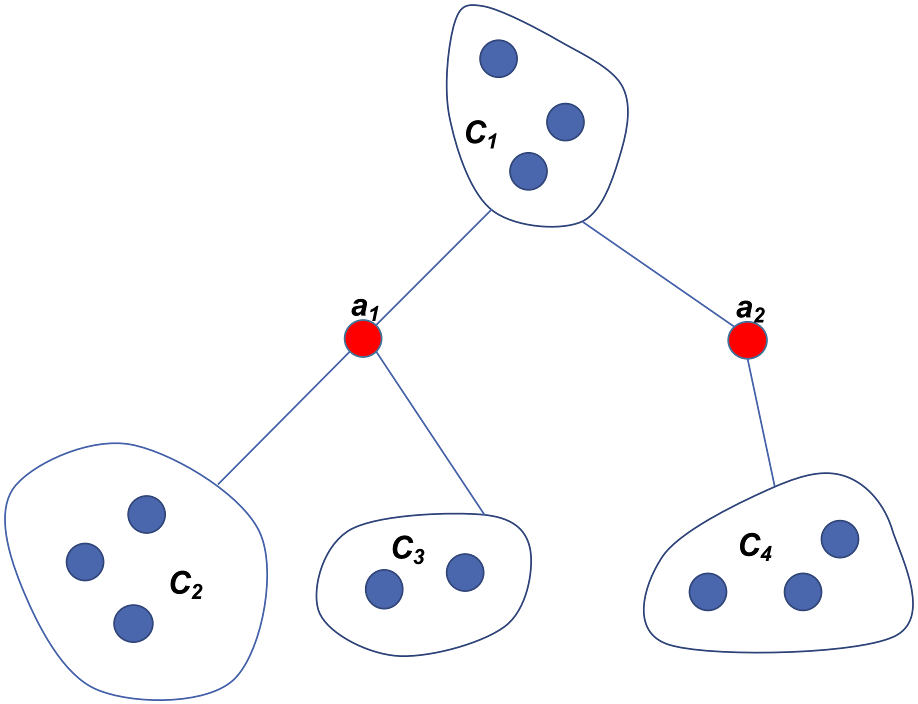
Edges joining two articulation points (e_C)

$$\geq 2^{|CC(G)|} \prod_{\substack{C \in CC(G) \\ |AP(C)|>0}} 2(1 + |AP(C)|)$$

GECCO 2018: C., Ochoa, Whitley, Tinós

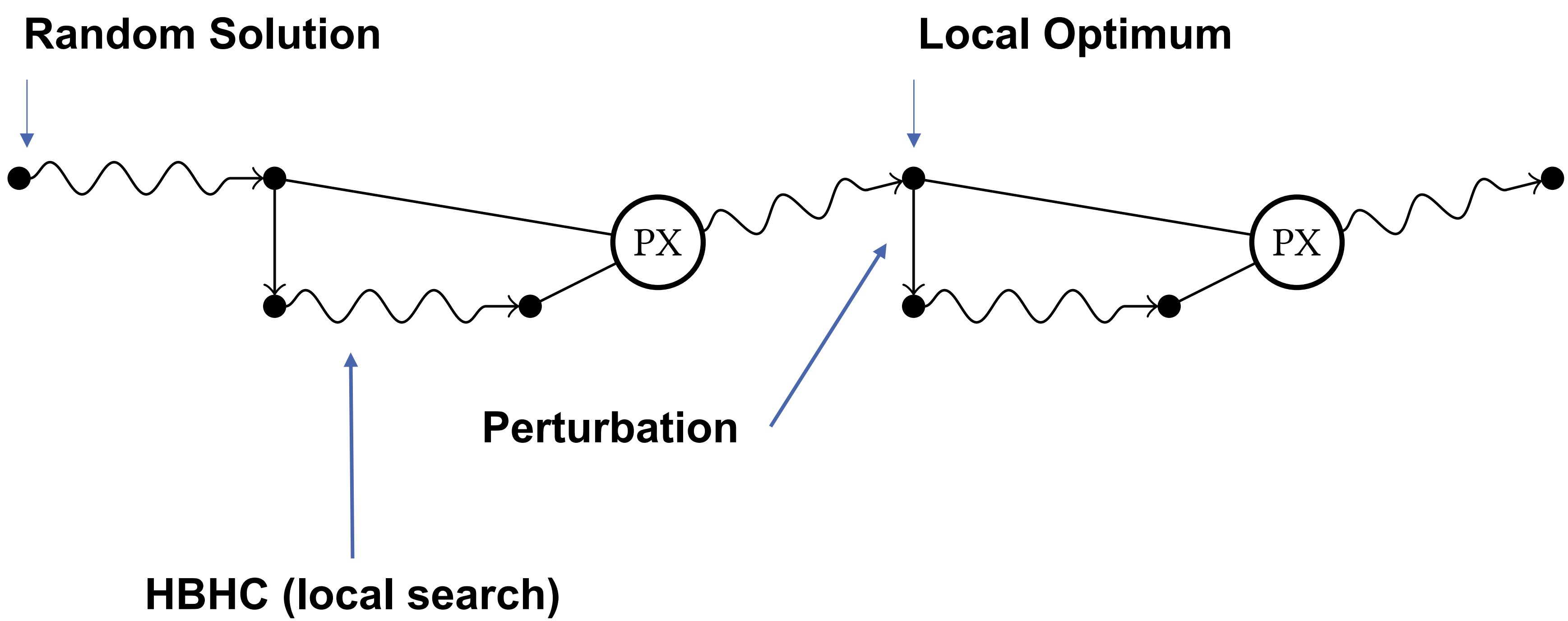
Articulation Points Partition Crossover (APX)

All the analysis can be done using Tarjan's algorithm to find articulation points (DFS-like algorithm) : time complexity is the same as the original PX



GECCO 2018: C., Ochoa, Whitley, Tinós

Deterministic Recombination and Iterated Local Search (DRILS)

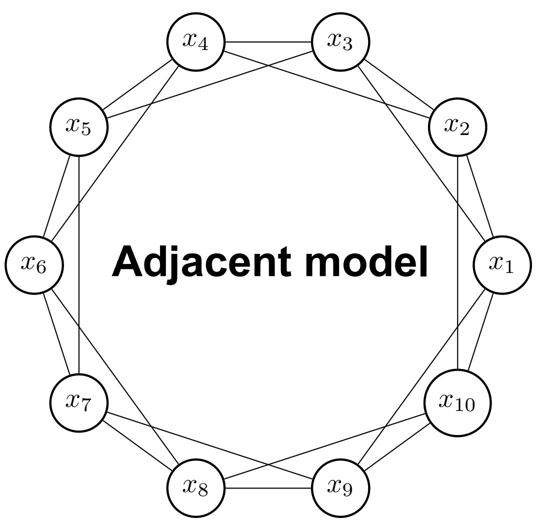


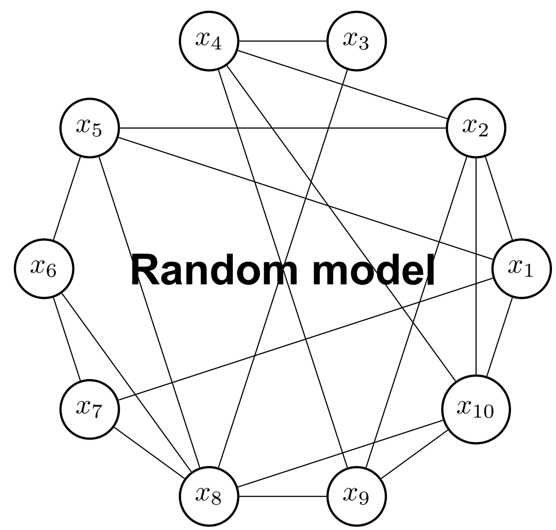
Experimental Results

- **An NK Landscape is a pseudo-Boolean optimization problem with objective function:**

$$f(x) = \sum_{l=1}^{N} f^{(l)}(x)$$

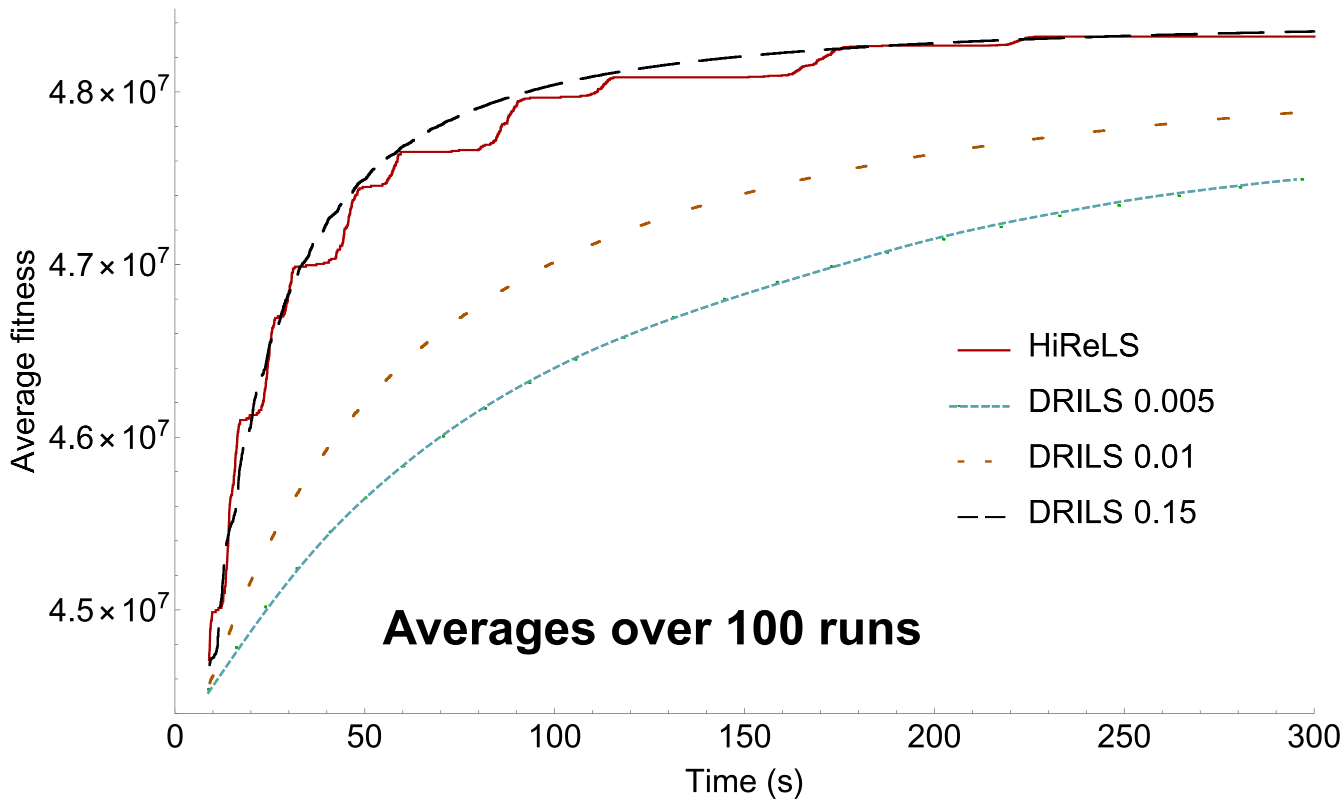
where each subfunction $f^{(l)}$ depends on variable x_l and K other variables





Experimental Results

1M variable adjacent NK Landscape with K=3



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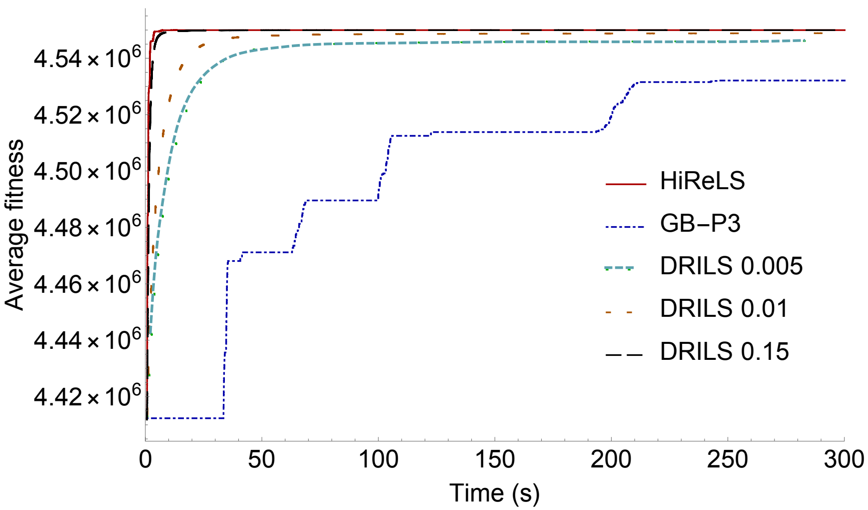
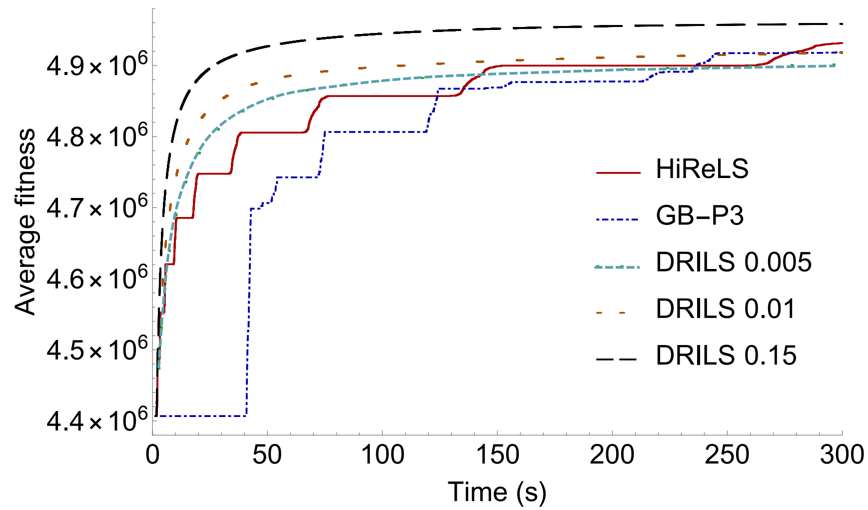
Experimental Results

100,000 variable adjacent NK Landscape



K=1



K=5

GB-P3: Gray-Box Parameter-less Population Pyramid

GECCO 2015: Goldman, Punch

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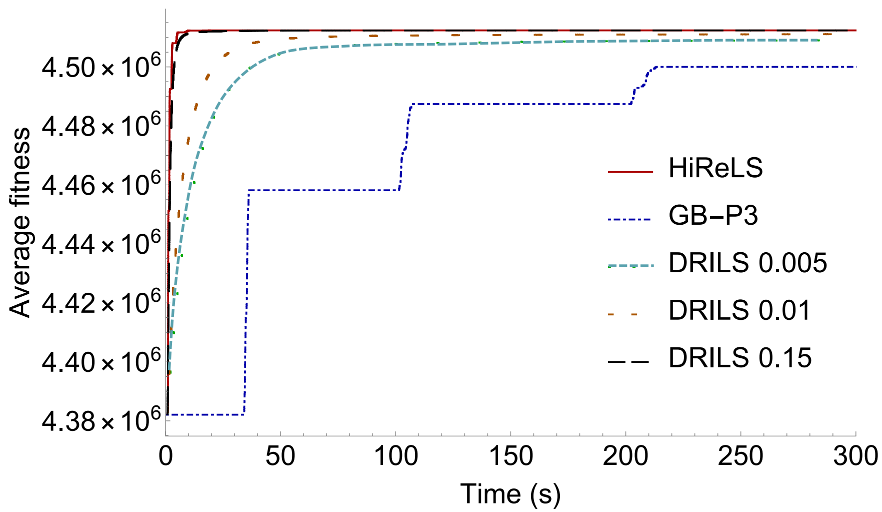
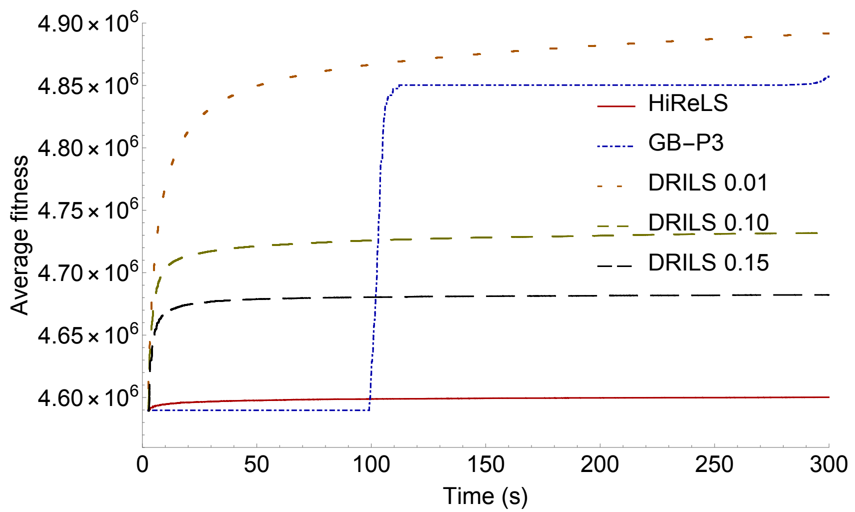
Experimental Results

100,000 variable random NK Landscape



K=1



K=5

Experimental Results

Average number of components found by Partition Crossover

	Perturbation Factor (α)				
K	0.005	0.01	0.05	0.10	0.15
1	683	1,314	6,059	11,442	16,259
2	967	1,772	6,938	11,426	13,428
3	1,041	1,810	4,970	3,639	2,367
4	993	1,657	1,780	661	301
5	903	1,344	517	100	38

$2^{4,970}$ solutions considered in each PX ($10^{1,485}$ solutions per nanosecond)

1M variable random NK Landscapes

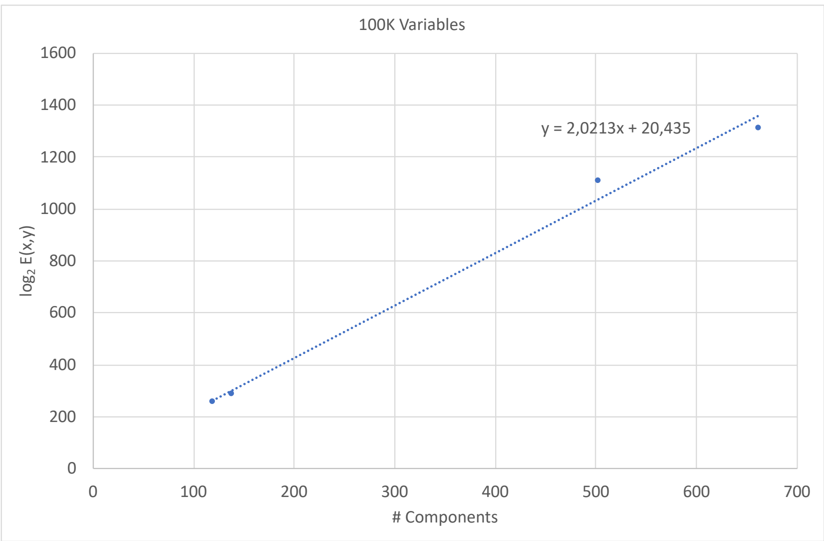
Experimental Results

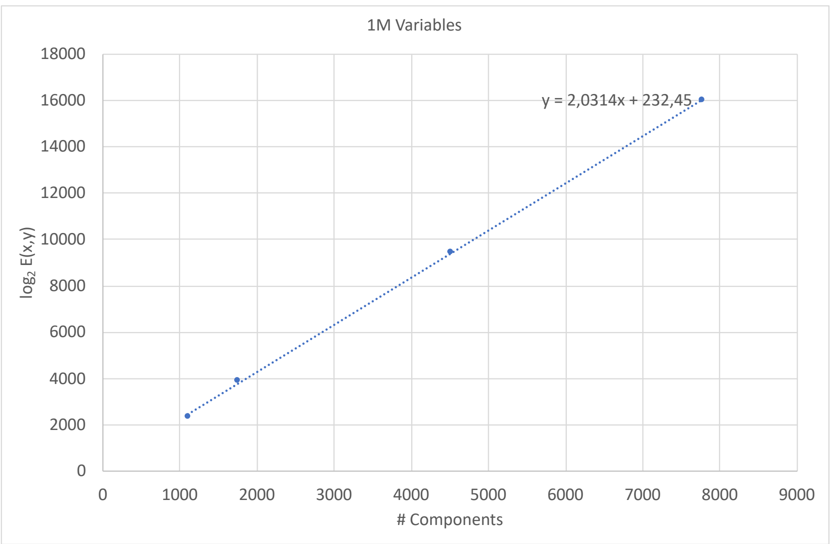
APX runtime is in the same order of magnitude than that of PX

N	K	#Comp.	#APs	d_a	$\log_2 E(x,y)$	Runtime (ms) APX	Runtime (ms) PX
10^5	2	662	687	2.25	1 311	55	46
	3	503	1 151	2.37	1 105	67	73
	4	138	196	2.33	286	55	52
	5	119	218	2.36	254	63	52
10^6	2	7 774	10 836	2.28	15 987	1 383	970
	3	4 515	21 793	2.35	9 454	1 785	2 485
	4	1 748	6 281	2.38	3 907	1 360	1 439
	5	1 105	7 207	2.34	2 341	1 633	1 559

Experimental Results

APX runtime is in the same order of magnitude than that of PX and the number of solutions explored is squared!





$$|APX| \approx |PX|^2$$

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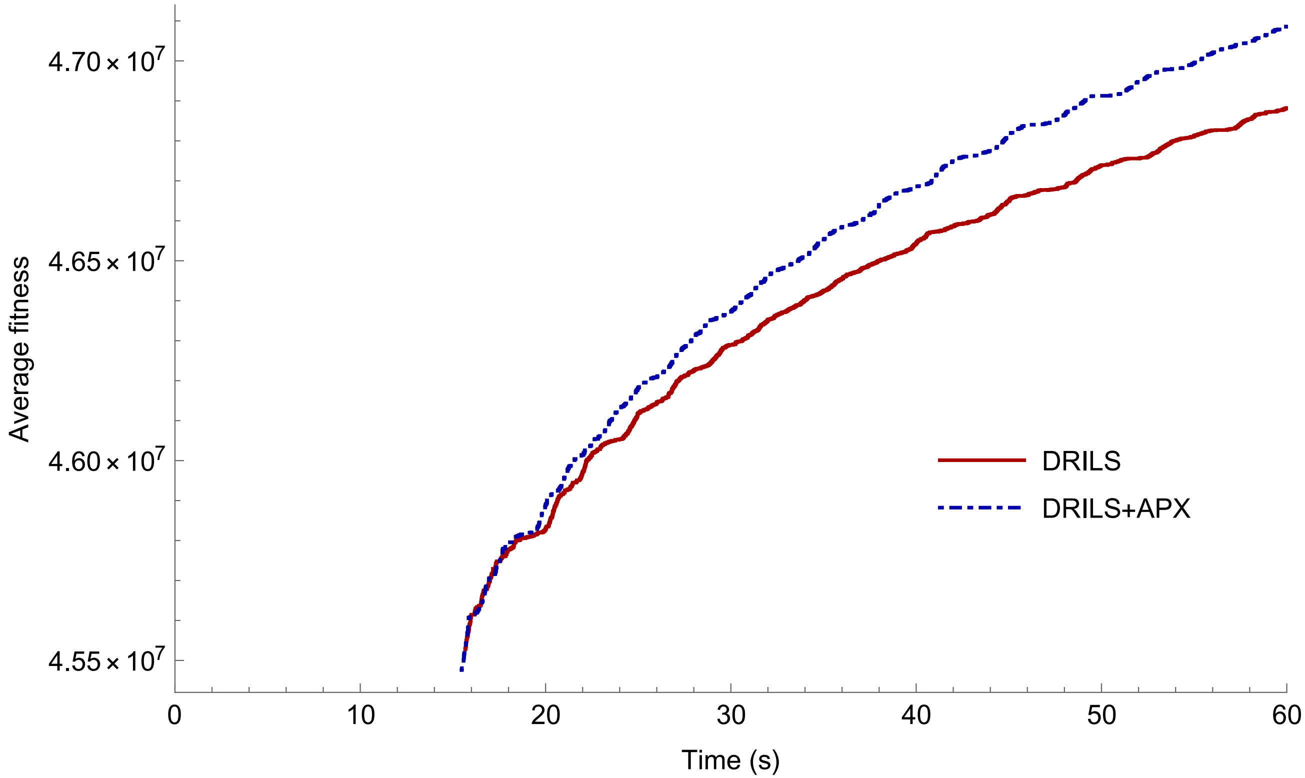
Experimental Results

DRILS and DRILS+APX solving NKQ Landscapes with N=1 Million and K=3



Experimental Results

DRILS and DRILS+APX solving MAX-SAT (instances from MAX-SAT Evaluation 2017)

		DRILS performance			Runtime (μs)	
Instances	α	APX	PX	Sim.	APX	PX
Unweighted	0.10	78	1	81	463	454
	0.20	82	2	75	684	729
	0.30	85	2	73	849	1 060
Weighted	0.10	26	19	87	1 425	882
	0.20	49	14	69	1 859	1 416
	0.30	77	5	50	2 365	1 713

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Conclusions

- The Variable Interaction Graph provides useful information to improve the search
- Articulation Points Partition Crossover squares the number of solutions considered by PX in around the same time
- APX is specially good in Unweighted MAX-SAT instances (many plateaus)
- Take home message: use Gray-Box Optimization if you can

Future Work

- Plateaus exploration in MAX-SAT guided by APX
- New ways of perturbing the solution to maximize the components in (A)PX
- Look at the Variable Interaction Graph of industrial problems

Enhancing Partition Crossover with Articulation Points Analysis

Francisco Chicano, Gabriela Ochoa, Darrell Whitley, Renato Tinos

ECOM

2018-07-18 10:40-11:05 Tersa Hall (1F)

Best Paper candidate

Acknowledgements

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Thanks for your attention!!!