

Solving large scale hub location problems by neural networks

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Facility Location

- **Classical facility location problems**

- p-median problems
- Uncapacitated facility location problems
- p-center problems
- Covering problems

- **Analogous hub location problems**

- p-hub median problems
- Uncapacitated hub location problems
- p-hub center problems
- Hub covering problems

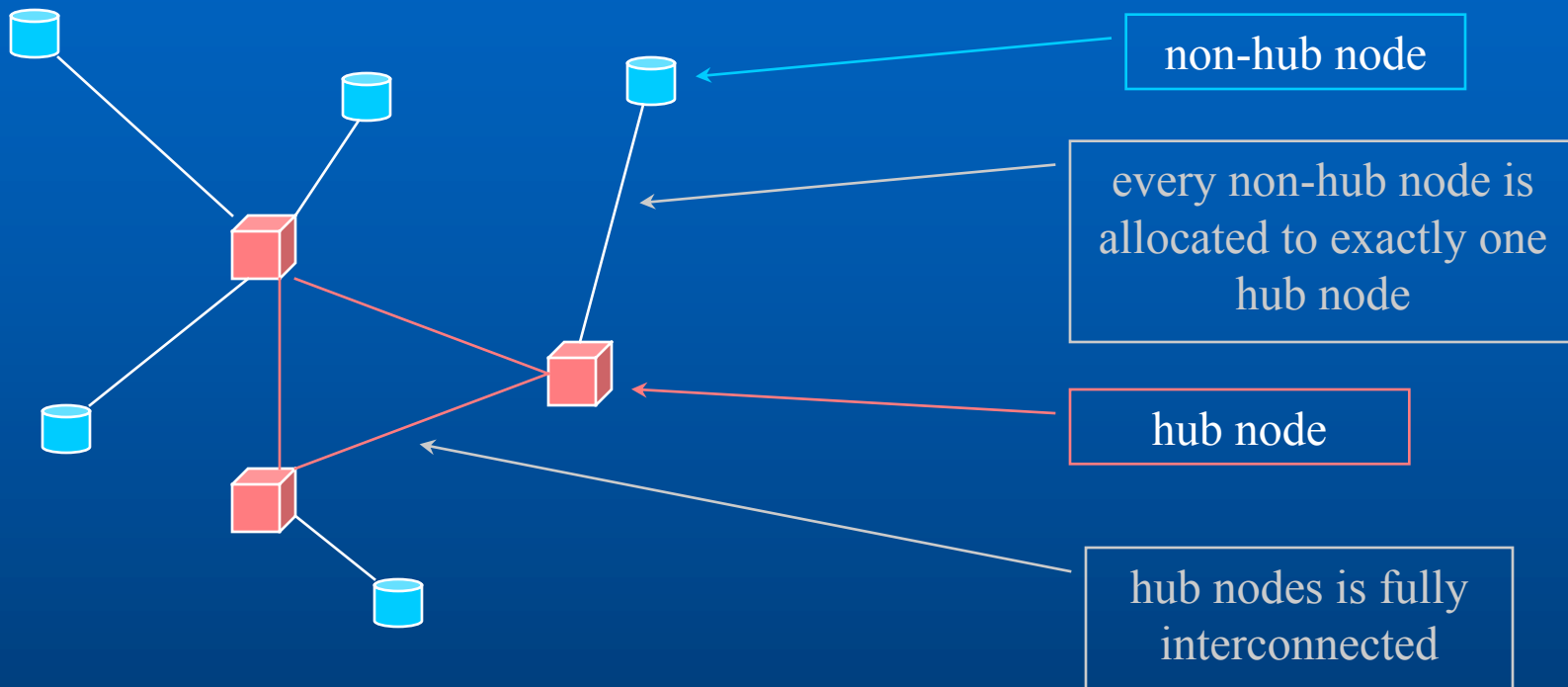
Difference

Single allocation
Multiple allocation

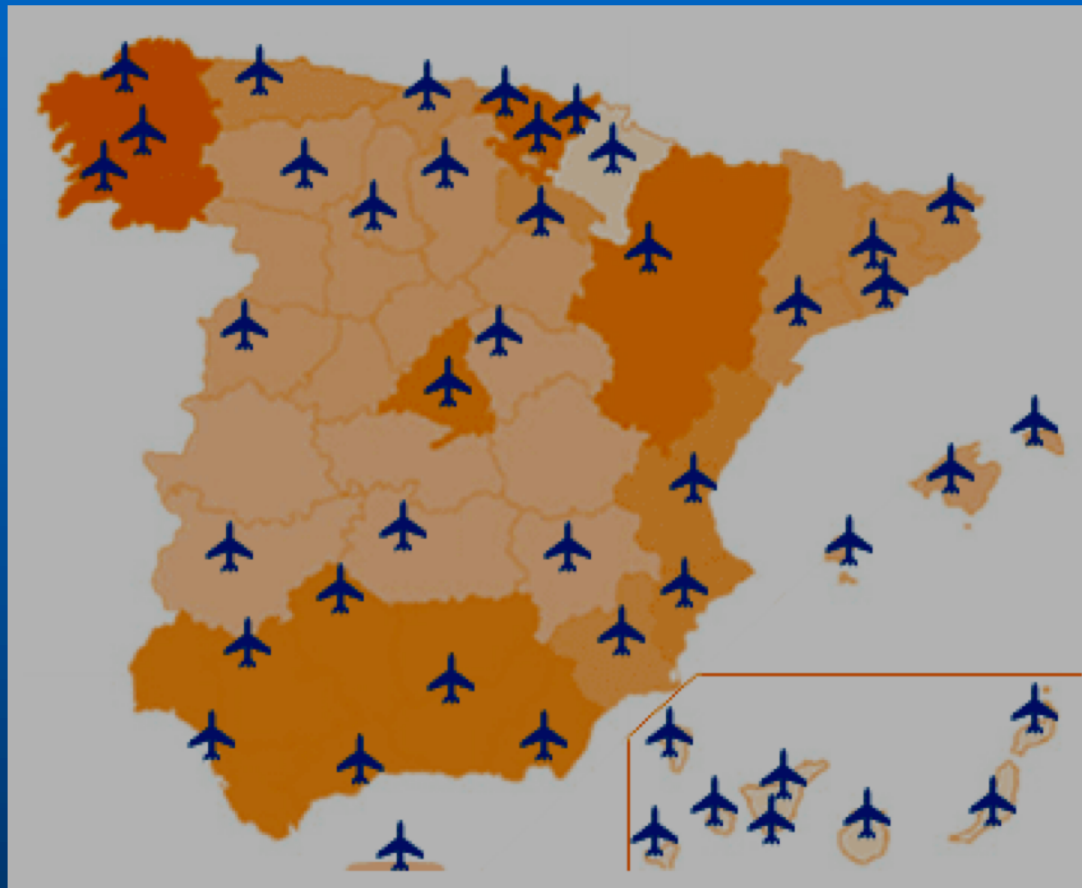
p-Hub Median Problem

- **Uncapacitated Single Allocation p -Hub Median Problem (USA p HMP)**
 - O'Kelly (1986)
 - NP-Hard
 - Numerous proposed procedures
 - Heuristics
 - Tabu search
 - Simulated Annealing
 - Modified Hopfield network
 - ...

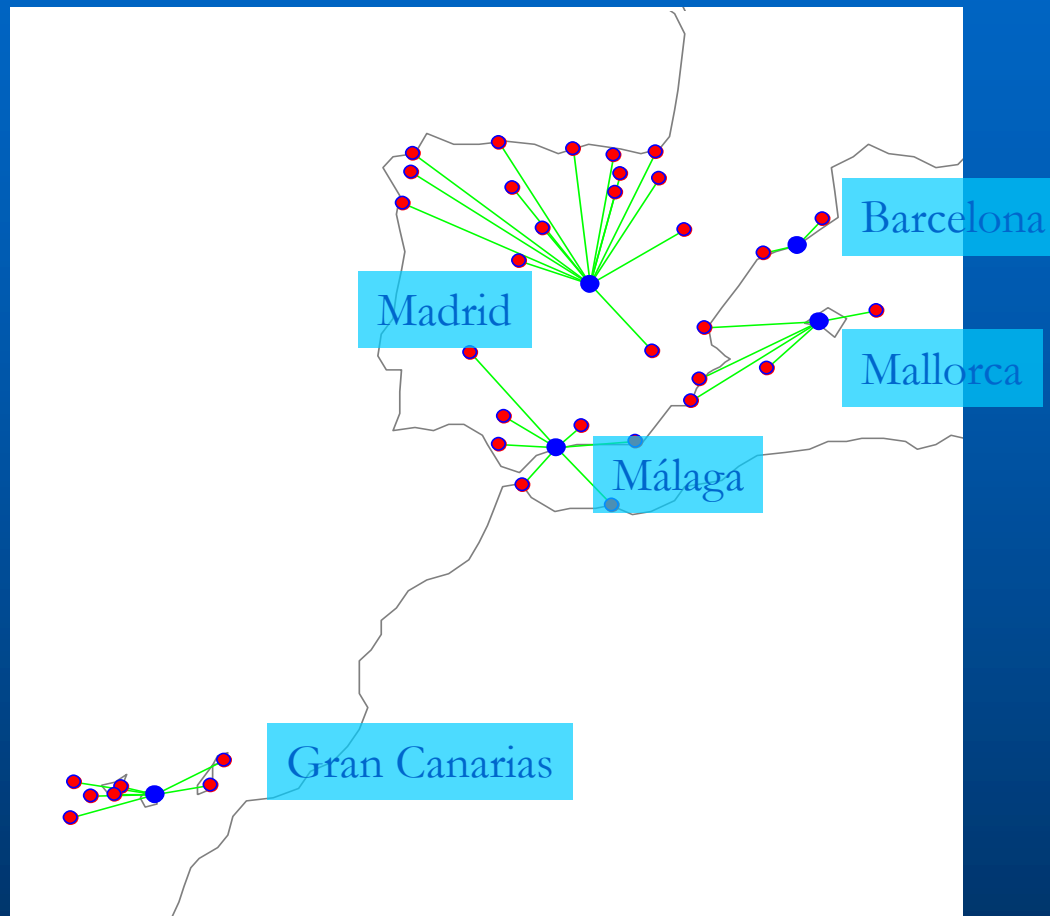
p-Hub Median Problem



Application



Application



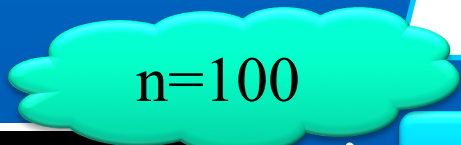





USApHMP Formulations

- **O'Kelly (1987)** | First formulation
- **Campbell (1994)** | $O(n^4)$ variables
- **Ernst and Khrisnamoorthy (1996)** | $O(n^3)$ variables
- **Ebery (2001)** | $O(n^2)$ variables

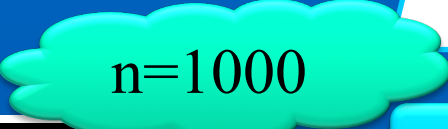
USApHMP Formulations

	Variables	Constraints
O'Kelly '87	n^2	$2n + 1$
Campbell '94	$n^4 + n^2 + n$	$n^4 + 2n^2 + n + 1$
Skorin-Kapov '96	$n^4 + n^2$	$2n^3 + n^2 + n + 1$
Ernst et al. '96	$n^3 + n^2$	$2n^2 + n + 1$
Ebery 2001	$3n^2 - 2n$	$2n^2 + 2n + 1$

USApHMP Formulations

	Variables	Constraints
 	10,000	201
	100,010,100	100,020,101
	100,010,000	2,010,101
	1,010,000	20,101
	29,800	20,201

USApHMP Formulations

	Variables	Constraints
 O'Kelly '86	1,000,000	2,001
Campbell '94	1,000,001,001,000	1,000,002,001,001
Skorin-Kapov '96	1,000,001,000,000	2,001,001,001
Ernst et al. '96	1,001,000,000	2,001,001
Ebery 2001	2,998,000	2,002,001

Reduced Formulation

Minimize
$$\sum_{i=1}^n \sum_{j=1}^n \sum_{q=1}^p \sum_{k=1}^n \left[\beta w_{ik} c_{ij} + \gamma w_{ki} c_{ji} + \alpha \sum_{m=1}^n \sum_{r=1}^p w_{ik} c_{jm} x_{kr} y_{mr} \right] x_{iq} y_{jq}$$

Subject to:

$$\sum_{q=1}^p x_{iq} = 1 \quad i = 1, 2, \dots, n$$

$$\sum_{j=1}^n y_{jq} = 1 \quad q = 1, 2, \dots, p$$

Number of points (nodes)

Number of facilities (hubs)

Transportation cost associated between nodes i and j

Amount of flow from the node i to the node k

Node i is allocated to the cluster q ?

Node j is the hub in the cluster q ?

Reduced Formulation

Outgoing
cost
(node → hub)

Incoming
cost
(hub → node)

Inter-hub
cost
(hub ↔ hub)

Minimize
$$\sum_{i=1}^n \sum_{j=1}^n \sum_{q=1}^p \sum_{k=1}^n \left[\beta w_{ik} c_{ij} + \gamma w_{ki} c_{ji} + \alpha \sum_{m=1}^n \sum_{r=1}^p w_{ik} c_{jm} x_{kr} y_{mr} \right] x_{iq} y_{jq}$$

Subject to:

$$\sum_{q=1}^p x_{iq} = 1 \quad i = 1, 2, \dots, n$$

$$\sum_{j=1}^n y_{jq} = 1 \quad q = 1, 2, \dots, p$$

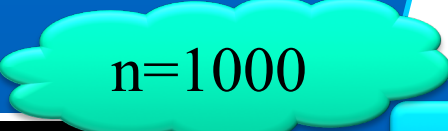
Each node is
allocated to

Statistics

$2np$ binary variables
 $n+p$ equality constraints

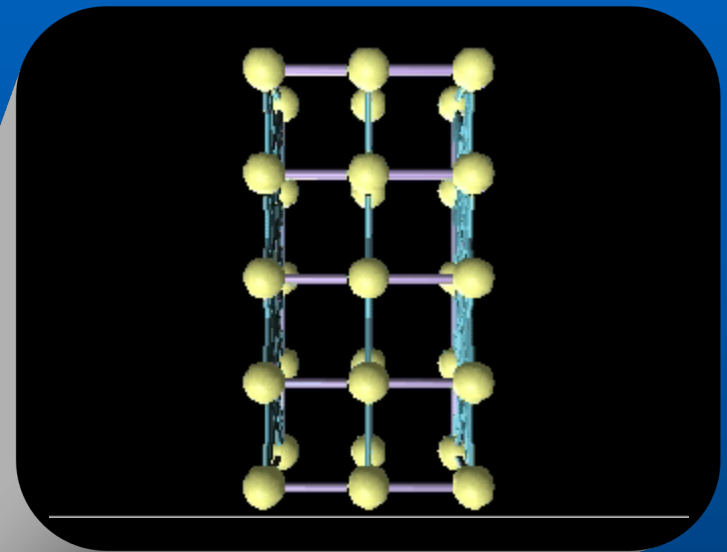
hub

USApHMP Formulations

	Variables	Constraints
 O'Kelly '87	1,000,000	2,001
Campbell '94	1,000,001,001,000	1,000,002,001,001
Skorin-Kapov '96	1,000,001,000,000	2,001,001,001
Ernst et al. '96	1,001,000,000	2,001,001
Ebery 2001	2,998,000	2,002,001
Dominguez et al.	$2,000p$	$1,000 + p$

NEural LOCator (NELOC)

- **Competitive recurrent neural network**
 - **Two layers**
 - Allocation layer
 - Location layer
 - **Binary neurons**
 - **Applied to p -median**
 - C&OR 2008



Competitive Recurrent Neural Network model

- **Allocation neurons**

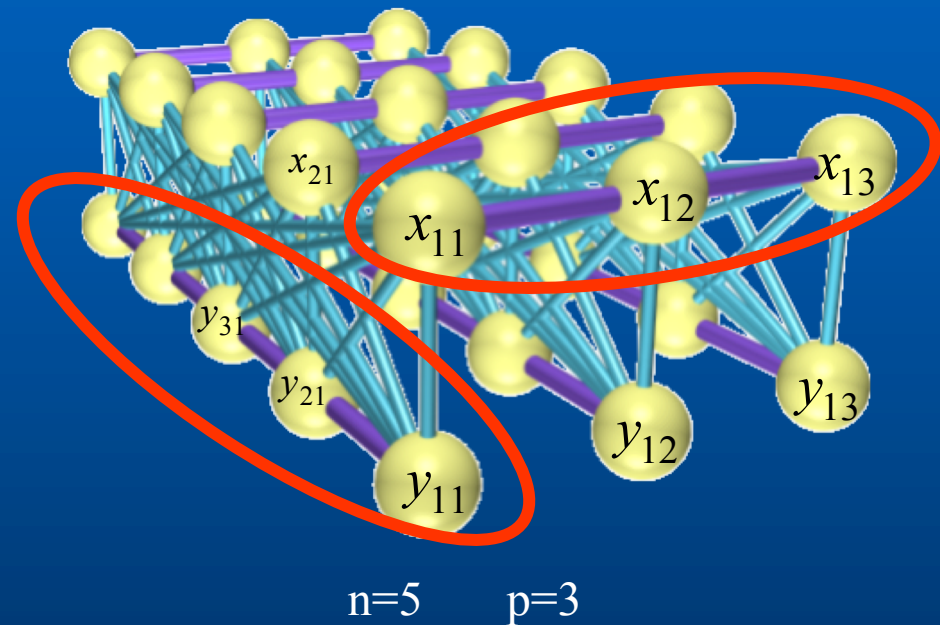
- *Allocate non-hub nodes with clusters*

- **Location neurons**

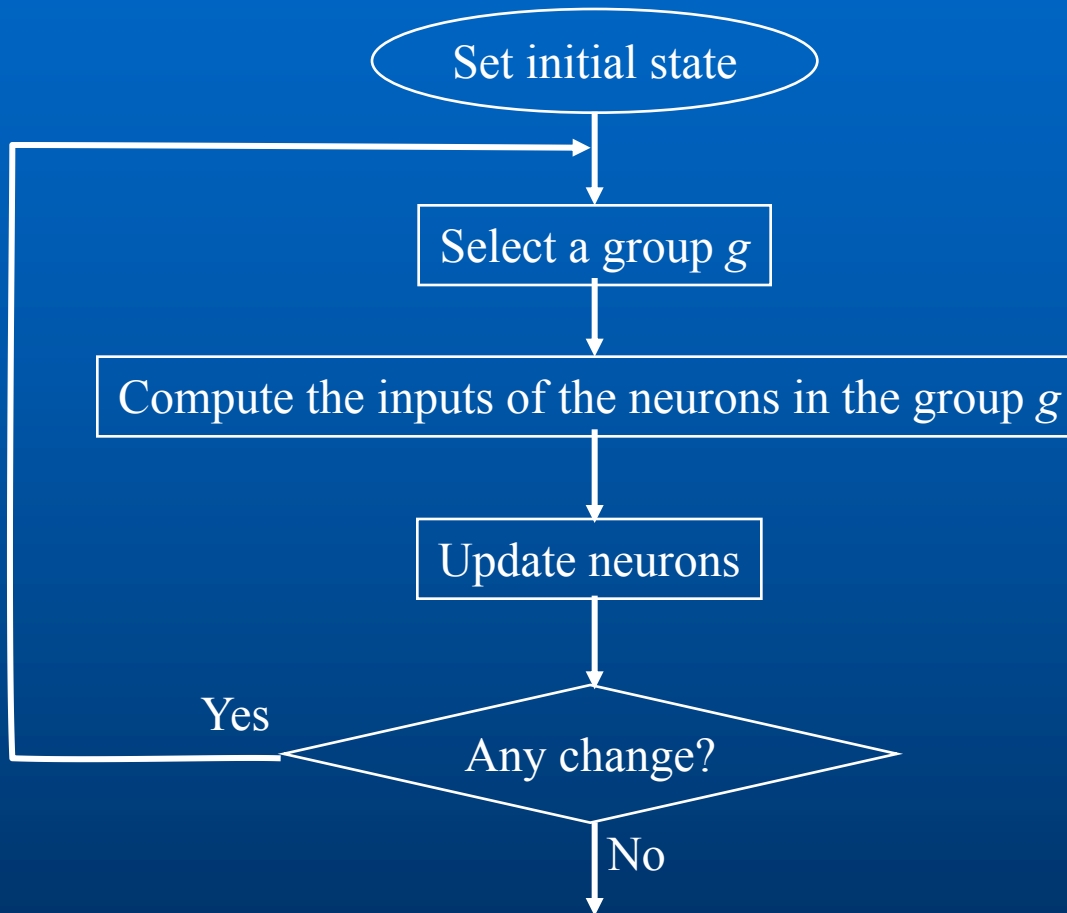
- *Locate hub nodes in clusters*

- **Competitive neuron groups**

- *Only one neuron is active in the group*

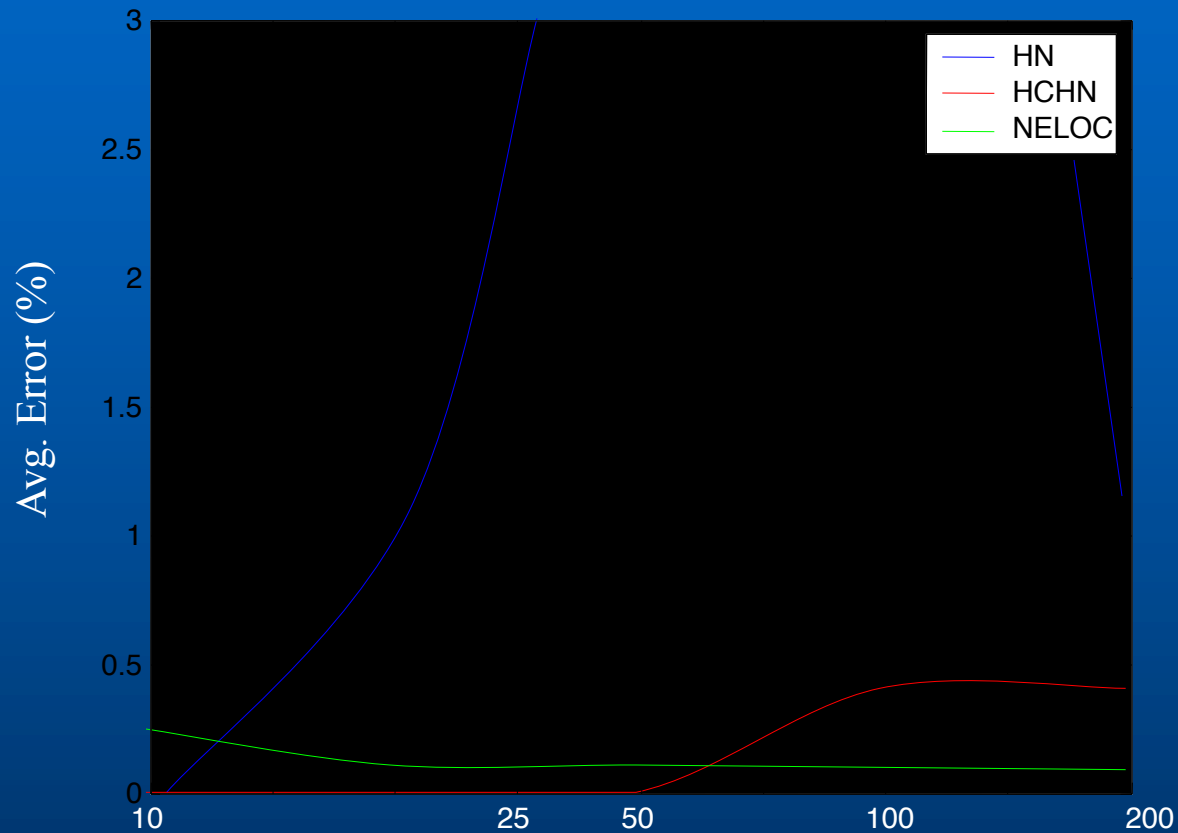


Neural Network Algorithm



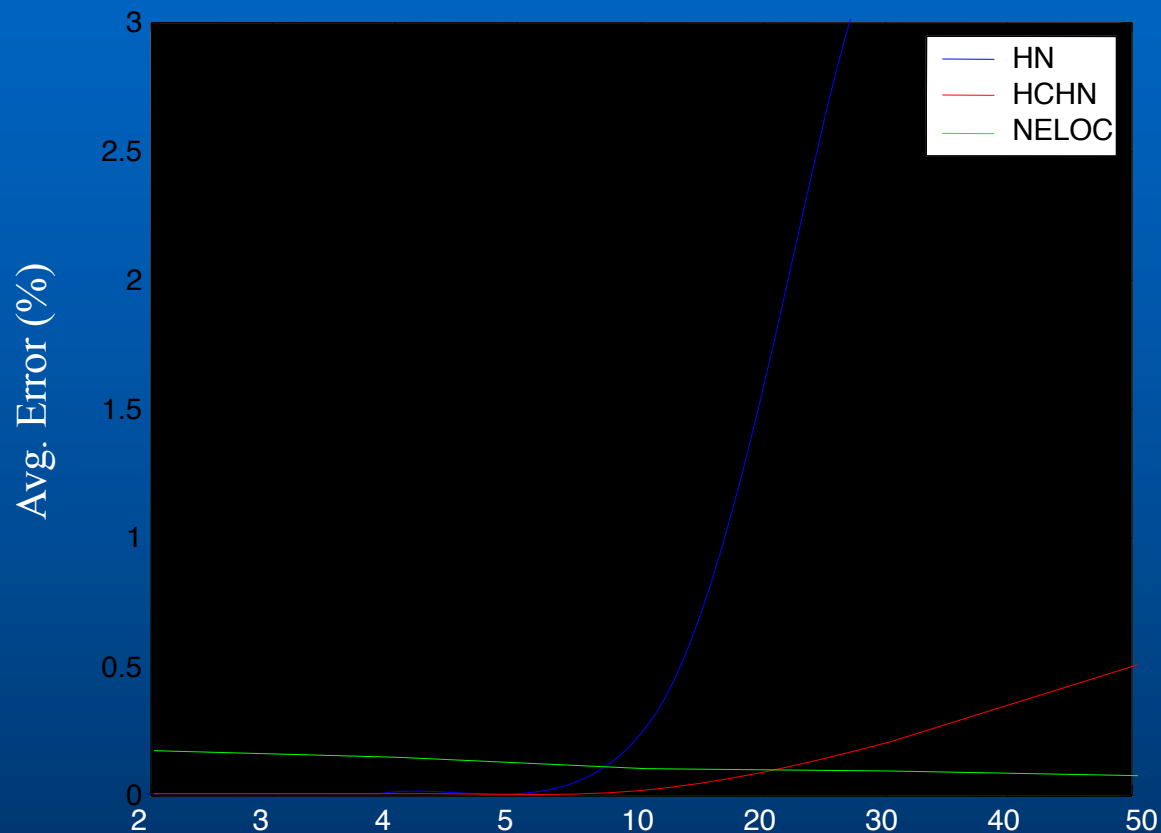
Experimental Results

Results for CAB & AP data sets



Experimental Results

Results for FL1400 data set



Concluding Remarks

- **NELOC can generate good solutions for location problems**
- **New reduced formulation for the p -hub median problem**
- **Competitive dynamics**
 - Feasible solutions
 - Convergence
- **Avoid typical parameters tuning of the Neural Networks**
- **No parameters to determine (temperature, term memory...)**

Further Works

- **Analyze the impact factor of the proposed model**
- **Compare results on large-scale problems with other heuristics**
- **Other hub location problem types**
 - **Fixed costs**
 - **p-hub center**
 - **Hub covering**